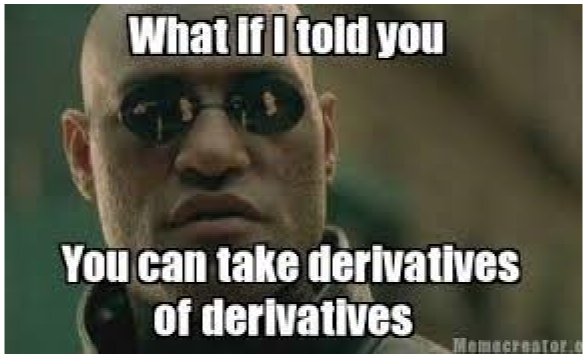
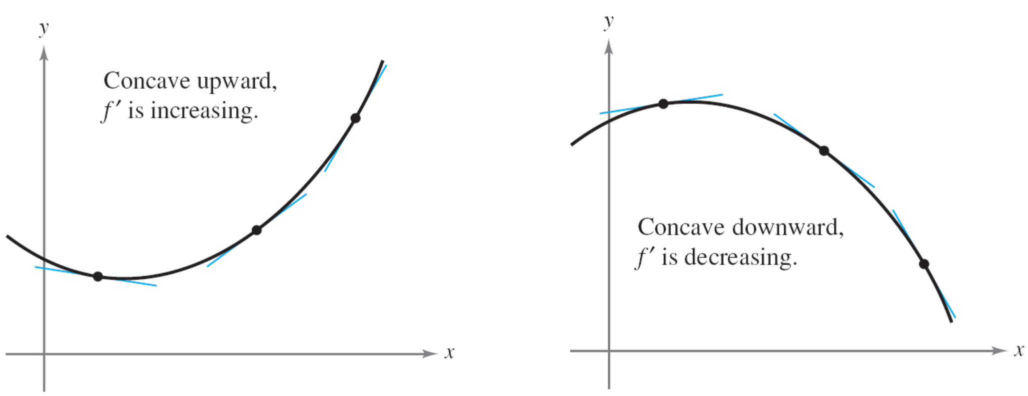
Calculus
Lesson 3.4: Concavity and the Second Derivative Test
Mrs. Snow, Instructor



In this section, we will see how locating the intervals in which f' increases or decreases can be used to determine where the graph of f is curving upward or curving downward.

DEFINITION OF CONCAVITY

Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.



(a) The graph of f lies above its tangent lines.

(b) The graph of f lies below its tangent lines.

THEOREM 3.7 TEST FOR CONCAVITY

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Determining Concavity

Determine the open intervals on which the graph is concave up or concave down.

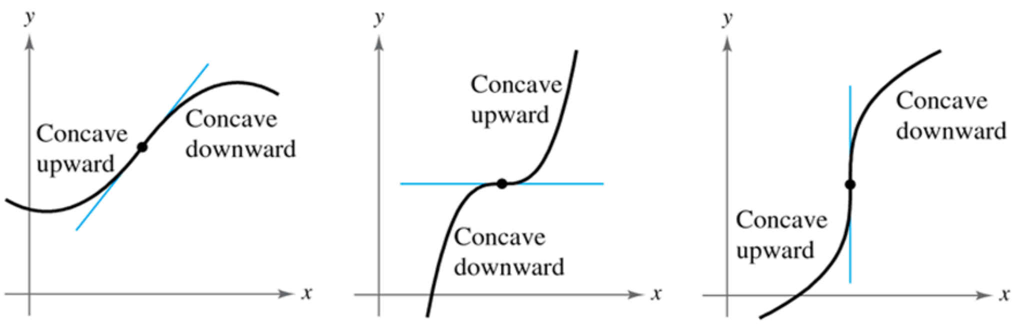
$$f(x) = \frac{6}{x^2 + 3}$$

Determine the open intervals on which the graph is concave up or concave down.

$$f(x) = \frac{x^2 + 1}{x^2 - 4}$$

DEFINITION OF POINT OF INFLECTION

Let f be a function that is continuous on an open interval and let c be a point in the interval. If the graph of f has a tangent line at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f if the concavity of f changes from upward to downward (or downward to upward) at the point.



The concavity of f changes at a point of inflection. Note that a graph crosses its tangent line at a point of inflection.

THEOREM 3.8 POINTS OF INFLECTION

If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c) = 0$ or f'' does not exist at $x = c$.

Determine the points of inflection and discuss the concavity of the graph.

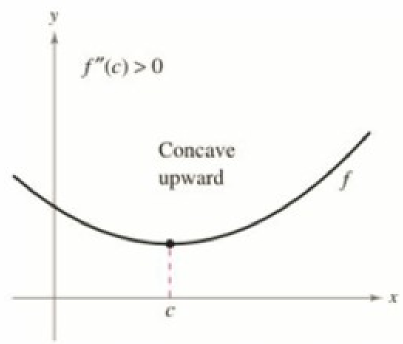
$$f(x) = x^4 - 4x^3$$

THEOREM 3.9 SECOND DERIVATIVE TEST

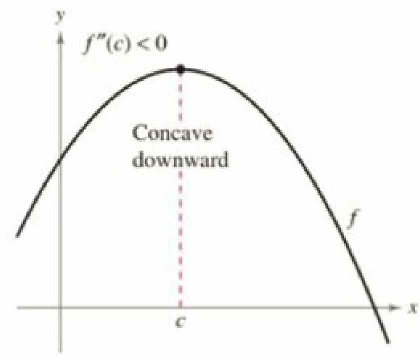
Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.



If $f'(c) = 0$ and $f''(c) > 0$, $f(c)$ is a relative minimum.



If $f'(c) = 0$ and $f''(c) < 0$, $f(c)$ is a relative maximum.

Find the relative extrema.

$$f(x) = -3x^5 + 5x^3$$