Algebra II
Lesson 9-6: Solving Rational Equations
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In this section we will learn techniques to solve for our variable when it is located in both the numerator and denominator. It is a fairly straight forward process, but the catch is that when multiplying an equation by an algebraic expression, there is a chance of getting **extraneous solutions.** So once again with the risk of extraneous expressions, we **must** check our solutions to verify that all are true solutions and/or the solution is not a restriction.

When there is only ONE fraction on each side of an equation, use cross-multiplication:

Solve:

$$\frac{4}{3x+3} = \frac{12}{x^2-1}$$

$\left(\frac{1}{4}\right)\cancel{4}(x^2-1) = \overset{3}{\cancel{12}}(3x+3)\left(\frac{1}{4}\right)$ factor out 4!

$x^2 - 1 = 3(3x+3)$

$x^2 - 1 = 9x + 9$

$\quad -9 \qquad -9$

Quadratic

$x^2 - 9x - 10 = 0$

$(x-10)(x+1) = 0$

$x = 10$ or $\cancel{x = -1}$

are these legitimate solutions? Restrictions?

1. take the cross product
2. using the distributive property simplify
3. isolate x terms and solve

$3x + 3 \neq 0$

$3x \neq -3$

$x \neq -1$

or

$x^2 - 1 \neq 0$

$x + 1 \neq 0 \quad x - 1 \neq 0$

$x \neq -1 \quad x \neq 1$

Try:

$$\frac{-4}{5(x+2)} = \frac{3}{x+2}$$

Restrictions:

$x + 2 \neq 0$

* $x \neq -2$

$-4(x+2) = (3)(5)(x+2)$

$-4x - 8 = 15x + 30$

$-19x = 38$

$\cancel{x = -2}$ *

NO solution

$\rightarrow \frac{-4\cancel{(x+2)}}{\cancel{(x+2)}} = \frac{15\cancel{(x+2)}}{\cancel{(x+2)}}$

$-4 \neq 15$

No sol.

(notice: Step 1, if we divide by factor $(x+2)$ we get a false statement ∴ no sol)

Equations with multiple terms on each side:

When the equation includes addition or subtraction of terms, or more than one fraction on one or both sides of the equations, there are a couple ways we can solve.

Multiply both sides of the equation with the LCD for all the denominators, this clears out all the fractions.

Solve:

Restrictions $x \neq 0, -1$

$$(x)(x+1)\left(\frac{4}{\cancel{x}} - \frac{3}{\cancel{x+1}}\right) = 1(x)(x+1)$$

$$4(x+1) - 3x = (x)(x+1)$$

$$4x+4-3x = x^2+x$$

Quadr.

$$0 = x^2 - 4$$

$$0 = (x+2)(x-2)$$

$$\underline{x=-2} \quad \underline{x=2}$$

1. find the LCD $(x)(x+1)$
2. multiply through by the LCD; <u>what you do the left, you do to the right.</u>
3. use distributive property to simplify
4. Isolate and solve for x.
5. verify that solutions are legit.

Option 2: Combine fractions on each side of the equation. Then the format is ready to do cross multiplication.

$$\frac{(x+1)}{(x+1)}\frac{4}{x} - \frac{3}{x+1}\frac{(x)}{(x)} = \frac{2}{x^2+x}$$

Restrictions $x \neq 0, -1$

$$\frac{4(x+1)-3x}{(x)(x+1)} = \frac{2}{x^2+x}$$

$$\frac{4x+4-3x}{x^2+x} = \frac{2}{x^2+x}$$

$$2(\cancel{x^2+x}) = (\cancel{x^2+x})(x+4)$$

$$2 = x+4$$

$$\underline{x=-2}$$

Restrictions $x \neq 0$

$$\left(\frac{5}{5}\right)\frac{1}{2x} - \frac{2}{5x}\left(\frac{2}{2}\right) = \frac{1}{2}$$

$$\frac{5-4}{10x} = \frac{1}{2}$$

$$\frac{1}{10x} = \frac{1}{2}$$

$$10x = 2$$

$$x = \frac{2}{10}$$

$$x = 1/5$$

Why do we have to solve these problems??? Because of Application Problems!!!

Carlos can travel 40 miles on his motorbike in the same time it takes Paul to travel 15 miles on his bicycle. If Paul rides his bike 20 mi/hr slower than Carlos rides his motor bike, find the speed for each bike.

1. Write the facts about each individual:

Carlos' speed= c Paul's speed= $c-20$

Carlos' distance=40 Paul's distance=15

2. What relationship is equal, that is, what is in common? time $\Rightarrow$ $s=\frac{d}{t} \rightarrow t=\frac{d}{s}$

3. Since we are talking speed and time and distance, what is the equation can we use?

$t=\frac{d}{s}$ $t_c = t_p$

Since the times are equal we know that the ratio of $\frac{d}{s}$ for the boys is equal too:

now set about to solve for c!

cross multiply[

solve for c and solve for p

Carlos Paul

$t_c = \frac{40}{c}$ $t_p = \frac{15}{c-20}$

$$\frac{40}{c} = \frac{15}{c-20}$$

$$15c = 40(c-20)$$

$$15c = 40c - 800$$

$$800 = 25c$$

$$\frac{800}{25} = c$$

$c = 32$ mph $\rightarrow$ $P = 12$mph

32-20

Time to do a Job:

Jason can clean a large tank at an aquarium in 6 hours. When Jason and Lacy work together, they can clean the tank in 3.5 hours. How long would it take Lacy to clean the tank if she works by herself?

- Look at their rates: $Jason's\ rate + Lacy's\ rate = combined\ rate$
- Jason's rate: 1 tank / 6 hr $\rightarrow \frac{1}{6}$
- Lacy's rate: 1 tank / h hr $\rightarrow \frac{1}{h}$ h = hours
- the complete job rate is 1 tank / 3.5 hr
- substitute the rates into the rate equation:
- solve for h $6h\left(\frac{1}{\cancel{6}} + \frac{1}{h}\right) = \left(\frac{1}{3.5}\right)6h$

$$h + 6 = \frac{6h}{3.5}$$

$$6 = \frac{6}{3.5}h - h$$

$$6 = .714h \Rightarrow \underline{h = 8.4 \text{ hours}}$$

- Now if the facts were given that Jason could clean the tank in 6 hours, Lacy could clean the tank in 8.4 hours how long would it take if both worked together? It is the same basic set up: Jason + Lacy = total

$$\frac{1}{6} + \frac{1}{8.4} = \frac{1}{t}$$ where t = total time

$$.286 = \frac{1}{t}$$

$$t = 3.5 \text{ hours}$$

One pump can fill a tank with oil in 4 hours. A second pump can fill the same tank in 3 hours. If both pumps are used at the same time, how long will they take to fill the tank?

one pump : 1 tank / 4 hr

2nd pump : 1 tank / 3 hr

t = total hour

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{t}$$

$$\frac{3}{12} + \frac{4}{12} = \frac{1}{t}$$

$$\frac{7}{12} = \frac{1}{t}$$

$$7t = 12$$

$$t = \frac{12}{7} = 1.71 \text{ hours}$$

$.71\ hr\left(\frac{60\ min}{1\ hr}\right)$

~43 min

(or 1 hr & 43 min)