Algebra II

Lesson 9-5: Adding and Subtracting Rational Expressions

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Last section we saw how we could apply techniques for multiplying and dividing fractions to rational expressions. Well, we also can apply addition and subtractions techniques to rational expressions. So what are the steps in adding two fractions?

$\frac{2}{3}+\frac{3}{4}+\frac{1}{6}$

3: 3

4: $2\cdot 2$

6: $2 \cdot 3$

LCM: need: 3, $2\cdot 2$ *have these*

LCM: $2\cdot 2\cdot 3 = 12$

$\frac{4}{4}\cdot\frac{2}{3}+\frac{3}{3}\cdot\frac{3}{4}+\frac{2}{2}\cdot\frac{1}{6}=\frac{8}{12}+\frac{9}{12}+\frac{2}{12}=\frac{19}{12}$

1. Find the least common multiple of the denominators. What is the smallest number that has all the as factors?

2. multiply each term by "1" so that they have common denominators

3. add and simplify

For rational expressions it is a bit more complicated in that finding the LCM involves <u>factorization of a polynomial</u>, but you have the tools to do this!

Find the least common multiple:

$$x^2-9 \quad and \quad x^2+6x+9$$

$(x+3)(x-3)$ $\qquad$ $(x+3)(x+3)$

need one of each $\qquad$ Have $\qquad$ need another

$(x+3)(x-3)(x+3)$

Add/subtract:

$$\frac{2}{2x-3}+\frac{4x}{2x-3}$$

Same denominator

Add numerators and place over a single denom.

$= \frac{2+4x}{2x-3}$

$$\left(\frac{2y}{2y}\right)\frac{2x+1}{x^2y}+\frac{3}{2y^2}\,\frac{xy}{xy}$$

need $\quad x^2 y \quad 2y$ and another y

LCD = $2x^2y^2$

To get LCD multiply by needed factors.

$= \frac{2y(2x+1)}{2x^2y^2}+\frac{3(xy)}{2x^2y^2}$

$= \frac{4xy+2y+3xy}{2x^2y^2}$

$= \frac{7xy+2y}{2x^2y^2}$

$$\frac{1}{x^2-4x-12}+\frac{3x}{4x+8}$$

$$\left(\frac{4}{4}\right)\frac{1}{(x-6)(x+2)} + \frac{3x}{4(x+2)}\frac{(x-6)}{(x-6)}$$

need $(x-6)(x+2)\ (4)$ ← Have

So:

$$= \frac{4}{4(x-6)(x+2)} + \frac{3x(x-6)}{4(x-6)(x+2)}$$

$$= \frac{4+3x^2-18x}{4(x-6)(x+2)}$$ Ans

$$\frac{x}{3x^2-9x+6}-\frac{2x+1}{3x^2+3x-6}$$

$3(x^2-3x+2)\quad 3(x^2+x-2)$

$$\frac{(x+2)}{(x+2)}\frac{x}{3(x-2)(x-1)} - \frac{2x+1}{3(x+2)(x-1)}\frac{(x-2)}{(x-2)}$$

need: $3(x-2)(x-1)\ (x+2)$

$$= \frac{x(x+2)}{3(x+2)(x-2)(x-1)} - \frac{(2x+1)(x-2)}{3(x+2)(x-2)(x-1)}$$

$$= \frac{x^2+2x \quad -(2x^2-4x+x-2)}{(3)(x+2)(x-2)(x-1)}$$

$-3x$

$$= \frac{x^2+2x-2x^2+3x+2}{3(x+2)(x-2)(x-1)}$$

$$= \frac{-x^2+5x+2}{3(x+2)(x-2)(x-1)}$$ Ans

Complex Fractions

A fraction can be further complicated by both the numerator and denominator being fractions. When this is the case we have a **complex fraction.**

#1: Find the LCD of all rational expressions

$$\frac{\left(\frac{1}{x}+3\right)xy}{\left(\frac{5}{y}+4\right)xy}$$

need xy

$$= \frac{\frac{\cancel{x}y}{\cancel{x}}+3xy}{\frac{5x\cancel{y}}{\cancel{y}}+4xy}$$

$$= \frac{y+3xy}{5y+4xy}$$ Ans

#2: Simplify the numerator and denominator

$$\frac{\frac{1}{x}+3\frac{x}{x}}{\frac{5}{y}+4\frac{y}{y}}$$

$$= \frac{\frac{1}{x}+\frac{3x}{x}}{\frac{5}{y}+\frac{4y}{y}} = \frac{\frac{1+3x}{x}}{\frac{5+4y}{y}} \div$$

fraction over a fraction

Keep-change-flip

$$\frac{(1+3x)}{(x)}\cdot\frac{(y)}{(5+4y)}$$

$$\frac{y+3xy}{5x+4xy}$$ Ans

$$\frac{\frac{1}{x}}{\frac{5}{y}} \quad \div$$

fraction over a fraction

keep - change - flip

$$\frac{1}{x} \cdot \frac{y}{5} = \frac{y}{5x}$$

Ans

$$\frac{-3}{\frac{5}{x}+y} \quad \frac{x}{x}$$

$$\frac{-3}{\frac{5+xy}{x}} \quad \div$$

$$\frac{-3}{1} \cdot \frac{x}{5+xy} =$$

$$\frac{-3x}{5+xy}$$

Ans

Example of a more advanced problem:

method 1:

$$\frac{\dfrac{x-2}{x}-\dfrac{2}{x+1}}{\dfrac{3}{x-1}-\dfrac{1}{x+1}}$$

$$\frac{\left(\dfrac{x-2}{x}\cdot-\dfrac{2}{x+1}\right)\cdot\big(x(x+1)(x-1)\big)}{\left(\dfrac{3}{x-1}-\dfrac{1}{x+1}\right)\cdot\big(x(x+1)(x-1)\big)}$$

$$\frac{(x+1)(x-1)(x-2)-x(x-1)(2)}{3x(x+1)-x(x-1)}$$

$$\frac{x^3-2x^2-x+2-2x^2+2x}{3x^2+3x-x^2+x}$$

$$\frac{x^3-4x^2+x+2}{2x^2+4x}$$

1. LCD for all the denominators:
$x(x+1)(x-1)$

2.Look at the big fraction: multiply the big fraction by "1" our common denominator; remember distribution!!

3. When we multiply each term, cancel out the common factors; you should get rid of the denominators in your numerator and the denominator in the denominator! Whew!

4. Simplify the numerator and denominator

5. Combine like terms and we're done

method 2: combine terms in numerator and denominator then flip and multiply

$$\frac{x-2}{x}\frac{(x+1)}{(x+1)}-\frac{2}{x+1}\frac{x}{x}=\frac{x^2-3x-2}{x(x+1)}$$

$$\frac{3}{x-1}\frac{(x+1)}{(x+1)}-\frac{1}{x+1}\frac{(x-1)}{(x-1)}=\frac{2x+4}{(x-1)(x+1)}$$

$$\frac{\dfrac{x^2-3x-2}{x(x+1)}}{\dfrac{2x+4}{(x-1)(x+1)}}(\div)$$

$$\frac{x^2-3x-2}{x(x+1)}\cdot\frac{(x-1)(x+1)}{2x+4}$$

$$\frac{x^3-4x^2+x+2}{2x^2+4x}$$

Here we focus on the numerator separately from the denominator.

1. Find the LCD for the terms in numerator

2. Multiply and simply
3. Do the same for the denominator

4. combine and then do the flip-switch-multiply

One way may seem easier to some students and the other say easier for others. You choose, but remember, you also need to follow directions!