Algebra II

Lesson 9.3: Rational Functions and Their Graphs

Mrs. Snow, Instructor

The functions studied in the previous sections are members of a bigger group of functions called **rational functions**. A rational function is a ratio of two polynomial functions:

$$f(x) = \frac{P(x)}{Q(x)}$$

We will have both vertical and horizontal asymptotes with rational functions.

Vertical Asymptotes

When we look at a rational function realize there is a polynomial in the denominator. As we learned several chapters ago, the solutions to a polynomial are the values where $x = 0$. Now, the moment we put a polynomial in a denominator of a fraction, we are going to get numbers that will make the denominator equal to zero, and we all know that is a bad thing to have. Values that make the denominator equal to zero are restrictions on the domain of the rational function. The restrictions on the domain are described by the following definitions:

discontinuous: a break in the line or curve on a graph

point of discontinuity: the point at **x=a** where the function is undefined (point where the denominator =0).

vertical asymptote: the line formed by **x=a**, the curve will get closer and closer but never touch or cross this line. where denominator goes to 0

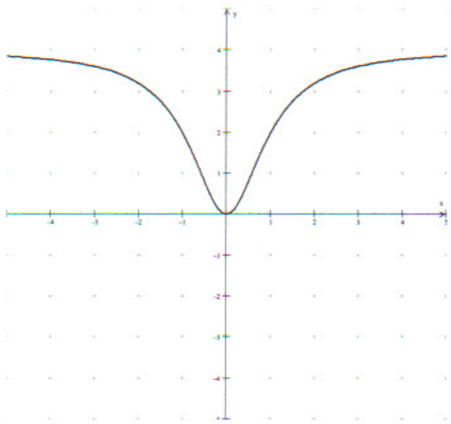
How do we find the points of discontinuity and thus the vertical asymptotes? Set the denominator equal to zero and solve for x. These values are the points of discontinuity and hence the vertical asymptotes.

Find the vertical asymptotes for: $y = \frac{2x}{3x^2+4} = 0$ $3x^2+4=0$ $3x^2=-4$ $x^2=-\frac{4}{3}$ what!? no VA	$y = \frac{x}{x^2-9}$ $x^2-9=0$ $\sqrt{x^2}=\sqrt{9}$ $x=\pm 3$ 2 Vertical Asymptotes	$y = \frac{3}{x^2-x-12}$ $x^2-x-12=0$ $(x-4)(x+3)=0$ $x=4 \quad x=-3$ 2 Vertical Asymptotes

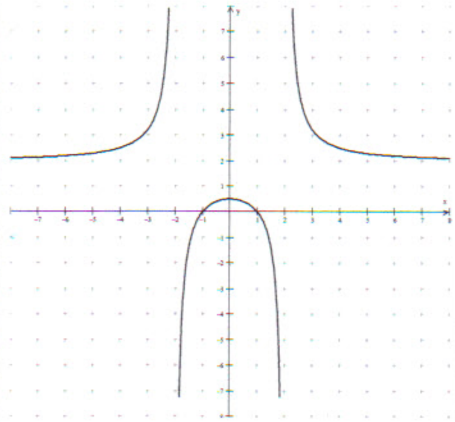
Some rational functions may not have any restrictions while others may have one or more, depending on the denominator.

Asymptotes or Holes ?

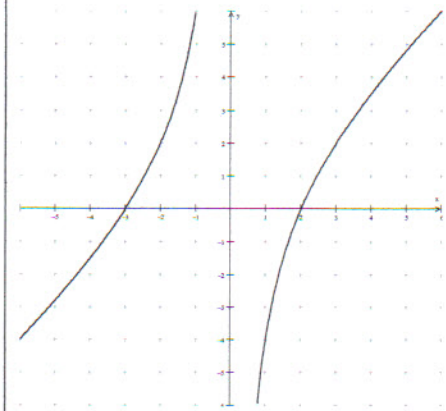
$\frac{4x^2}{x^2+1}$; **no restrictions on x**



$y = \frac{2x^2-2}{x^2-4}$ *where* $x \neq 2, -2$



$y = \frac{x^2+x-6}{x}$ *where* $x \neq 0$



Holes

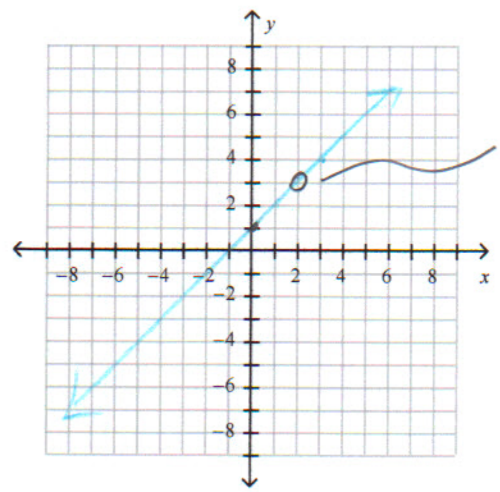
Graph: $y = \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} = x+1$

What does the graph look like?

Note that the (x-2) cancels out in the numerator and denominator. Check the table generated by the graph *what is the value of y for x=2?*

Error ⇒ point of discontinuity

When P(x) and Q(x) have a common real zero at a, then there is a hole in the graph. (the graphing calculator may not show the hole)



Hole at $x=2$
x-y point is hole at $(2,3)$

Describe the vertical asymptotes and/or holes of each rational function.

$y = \frac{x-2}{x^2+2x-3} = \frac{x-2}{(x+3)(x-1)}$ $x=-3 \quad x=1$ Vertical Asymptotes at $x=-3$ & $x=1$	$y = \frac{x^2-1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$ $x+1=0$ $x=-1$ Hole at $x=-1$

$y = \frac{x-2}{x^2+x-6} = \frac{(\cancel{x-2})1}{(x+3)(\cancel{x-2})}$ Vertical Asymptote at $x = -3$ Hole at $x = 2$ $x+3=0$ $x-2=0$	$y = \frac{x^2+3x}{x+3} = \frac{x(\cancel{x+3})}{(\cancel{x+3})}$ $x+3=0$ $x=-3$ Hole at $x = -3$

$x=-3$ $x=-2$

Horizontal asymptotes

The middle graph above shows a horizontal asymptote. *Why is it at y=2?* Note the following rules for horizontal asymptotes:

Degree of Polyn.

1. A rational function will have no more than one horizontal asymptote.
2. Degree of denominator is bigger than the degree of numerator: HA at $y = 0$ Bigger on Bottom
3. Degree of numerator is bigger than degree of denominator: No HA Bigger on Top
4. Degree of numerator = degree of denominator: divide leading coefficients: $y = \frac{LC}{LC}$ Exponents are the same

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LOOK AT THE EXPONENTS TO DETERMINE THE HORIZONTAL ASYMPTOTE:

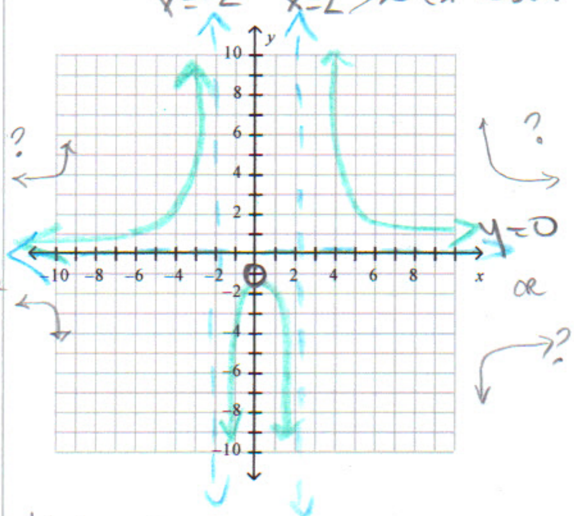
BIGGER ON BOTTOM: $0 = y$	BIGGER ON TOP: NONE	EXPONENTS ARE THE SAME: DIVIDE COEFFICIENTS

Find the horizontal asymptotes of the graph of each rational function

$y = \frac{3x - 4}{4x + 1}$ exponents are the same EATS DC $y = \frac{3}{4}$	$y = \frac{2x^2 + 5}{x + 1}$ BOTN no Horiz. Asym.	$y = \frac{x + 3}{(x - 1)(x - 5)}$ $\frac{x+3}{x^2 - 6x + 5}$ BOBO $y = 0$

When we know where and what type the points of discontinuity are, we can sketch graphs:

$$y = \frac{4x}{x^3 - 4x} = \frac{4x}{x(x^2-4)} = \frac{\cancel{4x}}{\cancel{x}(x+2)(x-2)}$$



Hole at $x=0$

VA at $x=2$, $x=-2$

HA BOBO $y=0$

Hole: $x=0 \Rightarrow$ graph

Sub $x=0$ $\quad \frac{4}{(\cancel{x^2}-4)} = \frac{-4}{4} = -1$

(0, -1

*Choose points left - middle - right

Look at the signs of the factors (+/-)

& then sign (+/-) of the polyn

$$\frac{4}{(x+2)(x-2)}$$

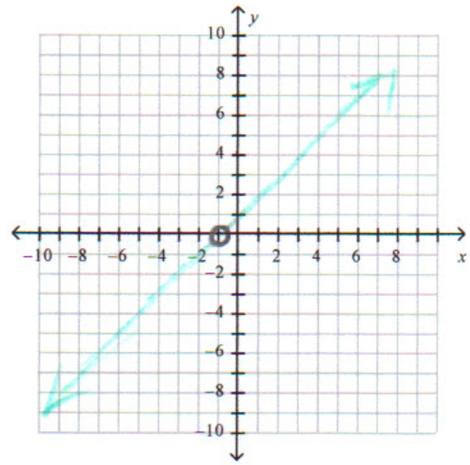
$x=-3: \frac{+}{(-)(-)} = +$

$x=+3: \frac{+}{(+)(+)} = +$

$x=-1: \frac{+}{(+)(-)} = -$

$x=+1: \frac{+}{(+)(-)} = -$

$$y = \frac{x^2+2x+1}{x+1} = \frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}} \Rightarrow y = x+1$$



Hole at $x=-1$ $\quad y=x+1$, $y=-1+1$, $y=0$