Algebra II
Lesson 8-5: Exponential and Logarithmic Equations
Mrs. Snow, Instructor

We have been looking at exponential equations and techniques to evaluate them, that is solve for x. We found that in a situation where for example: $3^x = 243$ we were able to get the right side into an exponential form and we found that $3^x = 3^5$. We also know a rule that says if the bases are equal the exponents must be equal; therefore $x = 5$.

Solve: $2^x = 30$

Plan A: In order to solve this equation, we will need x to be down on the ground floor. This can be done with logarithms, so we get: $log_2 30 = x$. Here we have the problem that we don't have the where with all to solve a base 2 log!!
Plan B: **Golden rule of algebra**: What you do to the left do to the right. This combined with our **power property** of logarithms will allow us to solve for x. So let's look back at our equation:

$2^x = 30$ $\log 2^x = \log 30$ $\frac{x \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 30}{\log 2}$ $x \approx 4.907$	1. Take the log of both sides 2. With the power property, take the exponent down in front of the log as a coefficient 3. Divide each side by the $log2$ Use a calculator to simplify
$6^{2x} = 21$ $\log 6^{2x} = \log 21$ $\frac{\cancel{2}x \cancel{\log 6}}{\cancel{2} \cancel{\log 6}} = \frac{\log 21}{2 \log 6}$ $x \approx .85$ Careful with calculator $\log(21)/(2\log(6))$	$3^{x+4} = 101$ $\log 3^{x+4} = \log 101$ $\frac{(x+4)\log 3}{\log 3} = \frac{\log 101}{\log 3}$ $x+4 = \frac{\log 101}{\log 3}$ $x = \left(\frac{\log 101}{\log 3}\right) - 4$ $x \sim .2$

$e^{2x+1} = 37$

base e use ln

$\ln e^{2x+1} = \ln 27$

$2x+1 \cancel{\ln e}^{1} = \ln 27$

$2x+1 = \ln 27$

$2x = \ln(27) - 1$

$x = \dfrac{\ln(27)-1}{2}$

$x \approx 1.305$

$10^x = 19$

$\log 10^x = \log 19$

$x \cancel{\log 10}^{1} = \log 19$

$x = \log 19$

$x \approx 1.279$

When **x** is part of the argument of a log function, a similar process may be used:

Solve: log jam! Break it up

$\log_{10}(3x+1) = 5$

$10^5 = 3x+1$

$10^5 - 1 = 3x$

$\dfrac{10^5 - 1}{3} = x$

$3333 = x$

1. Already in log form so make into an exponential function (base 10). **Recognize that by rewriting into exponential form, we can unlock the x and get it out of the log!!**
2. solve for **x** by isolating the **x** term and by clearing coefficient

$2\log x = -1$

$\log_{10} x^2 = -1$

$\frac{1}{10} = 10^{-1} = x^2$

$\sqrt{\frac{1}{10}} = x$

Square root $\pm$?? NO

$\dfrac{\sqrt{1}}{\sqrt{10}} = \dfrac{1}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} = +\dfrac{\sqrt{10}}{10}$

or $\approx .316$

$2\log x - \log 3 = 2$

Combine

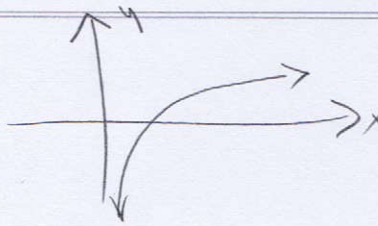
$\log x^2 - \log 3 = 2$

$\log_{10} \dfrac{x^2}{3} = 2$

$10^2 = \dfrac{x^2}{3}$

$(3)(100) = x^2$

$\sqrt{300} = x \approx 17.32$



only + on $\sqrt{\ }$

$\log_2(6x) - 3 = -2$ $+3$ $+3$

$\log_2 6x = 1$

$2^1 = 6x$

$\frac{2}{6} = x = \frac{1}{3}$

$\ln(3x) = 6$

$e^6 = 3x$

$\frac{e^6}{3} = x$

$134.48 \approx x$

CHANGE OF BASE

Now let's take a look at logs with bases other than 10. Evaluate $\boldsymbol{\log_3 29}$**,** there is no neat solution for this equation, unless we come up with another method. A method that works for these types of equations is called the **Change of Base Formula**.

Argument $\log_b M = \frac{\log_c M}{\log_c b}$

– the argument is kept in the numerator above the base in the denominator!
Or, the base stays in the basement! Whew!

Letting **c** be our base 10, we may take a ratio of the log of the argument to the log of the original base. So our
equation above becomes: $\log_3 29 = \frac{\log 29}{\log 3} = 3.065$

use base 10 or e
so we can use calulator!

$\log_2 7$

base 2 in basement!

$\log_2 7 = \frac{\log 7}{\log 2}$

≈ 2.81

use our evaluation tools:

$\log_2 7 = x$

$2^x = 7$

$\log 2^x = \log 7$

$x \log 2 = \log 7$

$x = \frac{\log 7}{\log 2}$:)

$\log_3 54$

Basement

$\frac{\log 54}{\log 3} \approx 3.63$

or

$\frac{\ln 54}{\ln 3} \approx 3.63$