Lesson Quadratic Inequalities

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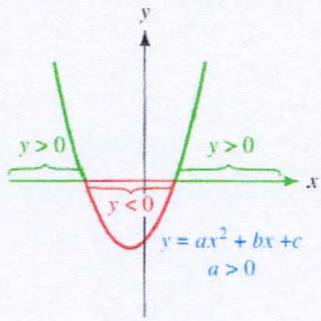
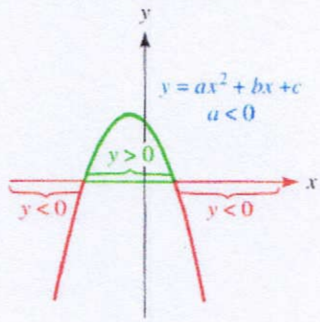
Suppose that $y = ax^2 + bx + c$. If the equal sign is replaced with an inequality we have what is called a **quadratic inequality**.

One-Variable Inequality:

- $ax^2 + bx + c > 0$ $(y > 0)$ The solution includes all x-values, where the graph of y is above the x-axis.
- $ax^2 + bx + c < 0$ $(y < 0)$ The solution includes all x-values, where the graph of y is below the x-axis.

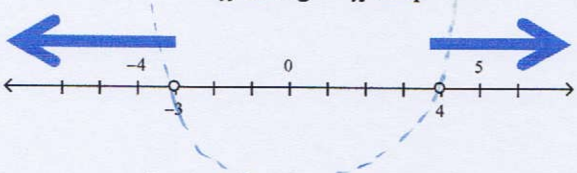




Example

Solve: $x^2 - x - 12 > 0$

$$x^2 - x - 12 = 0$$
$$(x + 3)(x - 4) = 0$$
$$x = -3 \quad x = 4$$



	-4	0	5
$x + 3$	$-$	$+$	$+$
$x - 4$	$-$	$-$	$+$
product	$+$	$-$	$+$

want > 0 *or product of factors will be* $+$

Answer:

$$x < -3 \text{ or } x > 4$$

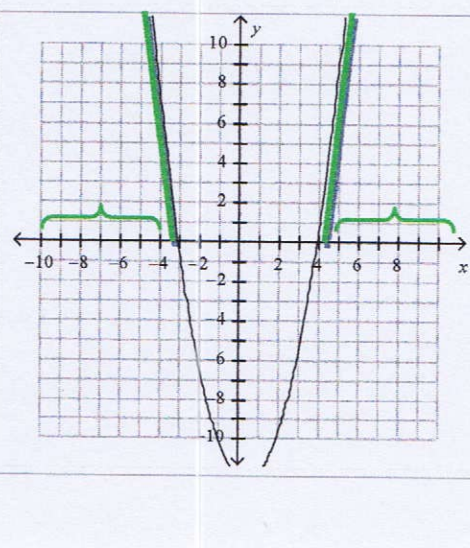
Interval notation: $(-\infty, -0) \cup (4, \infty)$

1. Replace inequality with an equal sign.
2. Factor the quadratic.
3. The solutions to the equation are the boundaries for the intervals that are the solutions to the inequality.
4. Make a number line, using the boundary numbers to separate sections of the number line.
5. Make a table of the factors vs. a test point on a number line to determine which segment(s) work for a solution for the inequality.
6. Solution segment(s) are where the test point yields correct sign for the product of the factors.

We want > 0 so the solutions are as show highlighted. With the > sign recognize that the end points are not part of the solution.

graphing the quadratic we see:

Where is the parabola above the x-axis?



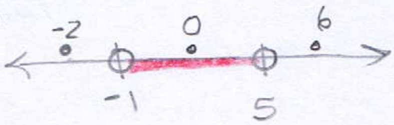
While the graphing is easy and thus visually we can see the solution areas. To better prepare us for precalculus, we need to understand how to solve inequalities algebraically.

Solve: $x^2 - 4x - 5 < 0$

$(x-5)(x+1) = 0$

$x = 5 \quad x = -1 \Leftarrow$ Boundaries & $\neq$

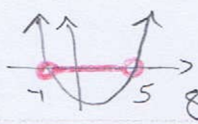
test points: $-2 \quad 0 \quad 6$



$(x-5)$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$
$(x-5)(x+1)$	$(-)(-)$ $+$	$(-)(+)$ $-$	$(+)(+)$ $+$

Solutions < 0 are negative so....

solution $-1 < x < 5$



graph

interval notation $(-1, 5)$

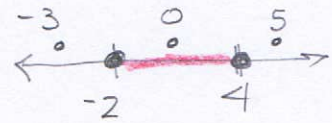
Solve: $x^2 - 2x - 8 \le 0$

$(x-4)(x+2) = 0$

$x = 4 \quad x = -2$

Boundaries are $x = -2$ & 4 & $=$

test points $-3 \quad 0 \quad 5$

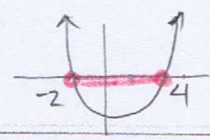


$(x-4)$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$
$(x-4)(x+2)$	$(-)(-)$ $+$	$(-)(+)$ $-$	$(+)(+)$ $+$

Solutions are < 0

$\therefore -2 \le x \le 4$

graph:



OR $[-2, 4]$

Solve: $-2x^2 - 6x + 20 \le 0$

factor out -2

$\left(-\frac{1}{2}\right) -2\,(x^2 + 3x - 10 \le 0)\left(-\frac{1}{2}\right)$

Multiply by neg!

$x^2 + 3x - 10 \ge 0$

$(x+5)(x-2) \ge 0$

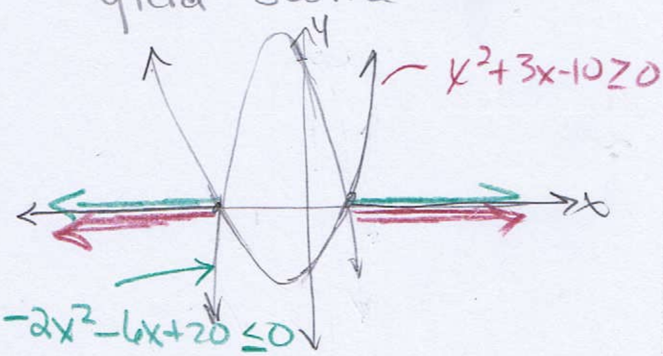
$x = -5 \quad x = 2$ Boundaries & $=$

test points $-6 \quad 0 \quad 3$

$x+5$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
$(x+5)(x-2)$	$(-)(-)$ $+$	$(+)(-)$ $-$	$(+)(+)$ $+$

Solution $(-\infty, -5] \cup [2, \infty)$

graphing we can see both original equation and factored equation yield same answer:



Applications

An object is launched at 4.9 meters per second from a 58.8-meter tall platform. The equation for the object's height at time t seconds after launch is $s(t) = -4.9t^2 + 4.9t + 58.8$, where s is in meters. When does the object hit the ground?

↑ time for a height $s = 0$

$0 = -4.9t^2 + 4.9t + 58.8$ ☺ factor out -4.9

$0 = \cancel{-4.9}(t^2 - t - 12)$

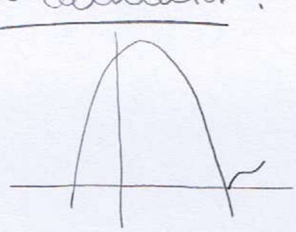
$0 = (t-4)(t+3)$

$t = 4$ $\quad t = \cancel{-3}$ no negative time

Ans: $t = 4$ sec.

Units required!!

with calculator:



[2nd] [trace] 2 (zero) [enter]

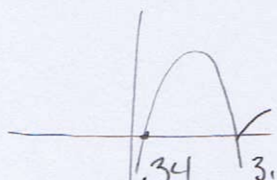
$x = 4$ ⟹ 4 seconds

An object is launched directly upward at 64 feet per second from a platform 80 feet high. The equation for the object's height is $h(t) = -16t^2 + 64t + 80$.

a) At how many seconds will the object have a height of 100 feet?

$-16t^2 + 64t + 80 = 100$ $\quad (-100, -100)$

$-16t^2 + 64t - 20 = 0$



[OR] * graph $y_1 = -16t^2 + 64t + 80$, $y_2 = 100$ and find intersections

Better, can see object motion

b) There are 2 answers. Why?

$x = .34$ sec

OR

$x = 3.66$ sec

Object is launched, goes up into the air passes 100 ft and comes down, passing 100 ft a second time.

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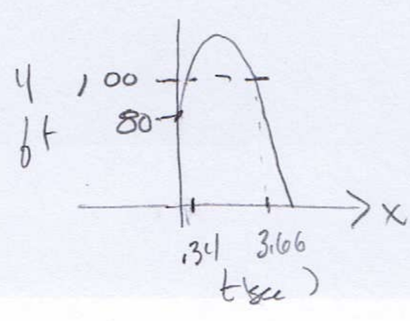
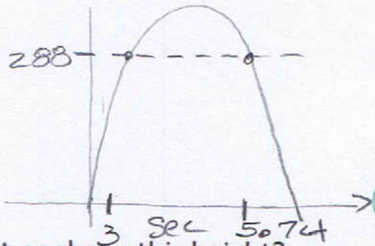


An object is launched from ground level directly upward at a rate of 144 feet per second. The equation for the object's height is $y = -16x^2 + 144x$.

a) What values of x is the object at **OR ABOVE** a height of 288 feet?

$y_1 = -16x^2 + 144x$

$y_2 = 288$



window: xmin = -10, ymin = -10

xmax = 10, ymax = 350

xScl = 1, yscl = 50

Object at or above 288 ft

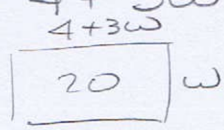
$3 \le x \le 5.74$ sec

b) How long is the object at or above this height?

$5.74 - 3 = 2.74$ sec

The area of a rectangle is 20 square inches. The length is 4 more than three times the width. Find the length and width of the rectangle. (Hint: draw a picture & set up a system of equations.)

$(l)(w) = \text{area}$ $\quad l = 4 + 3w$



$(4+3w)(w) = 20$

$4w + 3w^2 - 20 = 0$

$3w^2 + 4w - 20 = 0$ $\quad (3)(-20) = -60$

$3w^2 - 6w + 10w - 20 = 0$ $\quad (-6)(10) = -60$

$3w(w-2) + 10(w-2)$ $\quad -6 + 10 = +4$

$(3w+10)(w-2) = 0$ $\quad 4 + 3w = l$

$3w + 10 = 0$ $\quad w - 2 = 0$ $\quad 4 + 3(2) = 4 + 6 = 10$

$3w = -10$ $\quad w = 2$

$w = \frac{-10}{3}$ false

$w = 2$ in

$l = 10$ in