

## Algebra 2

### Lesson 5-7: Completing the Square

Mrs. Snow, Instructor

In this chapter, we have learned several ways to factor quadratic equations. Some equations that we say we cannot factor, may in fact be "forced" into a factorable form. This is accomplished by using a technique called **completing the square**.

First off, you will need to be comfortable using the factoring formula of the **Perfect square trinomials**:

$$a^2 + 2ab + b^2 = (a + b)^2 \text{ OR } a^2 - 2ab + b^2 = (a - b)^2$$

For example, factor:  $x^2 + 10x + 25$ ;

$$(x)^2 + 2(x)(5) + (5)^2$$

$$(x+5)(x+5) = (x+5)^2$$

Note:  $\frac{1}{2}$  of 10 is 5

from above:  $a$  is  $x$  and  $b$  is 5

1. rewrite the first and third terms as perfect squares
2. rewrite the linear term to verify that it is in fact in the form of  $2ab$
3. Factor

**Complete the square for each expression**  $\rightarrow \frac{1}{2}$  of linear coefficient  $\frac{1}{4}$  square it

$x^2 + 2x + \underline{\quad}$ $\frac{1}{2}(2) = 1 \quad 1^2 = 1$ <p><math>\rightarrow</math> check <math>(x+1)(x+1) \checkmark</math></p>	$x^2 - 12x + \underline{36}$ $\frac{1}{2}(-12) = -6 \quad (-6)^2 = 36$ <p><math>\rightarrow</math> check: <math>(x-6)(x-6) \checkmark</math></p>	$x^2 + 5x + \underline{\quad}$ $\frac{1}{2}(5) = \frac{5}{2} \quad \left(\frac{5}{2}\right)^2 = \frac{25}{4}$ $\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right)$
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So to complete the square, we force the equation into the perfect square trinomial form. Let's take a look at the example below and follow the steps:

$x^2 - 2x - 8 = 0$ $x^2 - 2x + \underline{1} = 8 + 1$ $\left(\frac{1}{2}\right)(-2) = -1 \quad (-1)^2 = 1$ <p>so:</p> $x^2 - 2x + 1 = 9$ $\sqrt{(x-1)^2} = \sqrt{9}$ $x-1 = \pm 3$ $x = 1 \pm 3$ $x = 1 + 3 = 4$ $x = 1 - 3 = -2$	<ol style="list-style-type: none"> <li>1. Move the constant (c) over to the right side</li> <li>2. clear out the quadratic term's coefficient (a) if <math>a \neq 1</math></li> <li>3. Take the coefficient of the linear term, <b>halve it, and square it</b>. Remember the sign of the linear term, will need it in a couple steps!</li> <li>4. Add this number to both sides of the equation (both sides so to keep the equation balanced).</li> <li>5. The left side is now a <b>perfect trinomial square</b>. Factor the left side of the equation. Linear term is negative so negative carries to the factored equation</li> <li>6. Now we can square root both sides of the equation to get rid of the radical.</li> <li>7. Solve for x and simplify the radical as needed</li> </ol> <p>Ans: <math>x = 4</math> or <math>-2</math></p> <p>check <math>\Rightarrow</math>  <math>4^2 - 2(4) - 8 = 0 \checkmark</math>  <math>(-2)^2 - 2(-2) - 8 = 0 \checkmark</math></p>
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Solve by completing the square

\* What you do to the left do to the right!

$$x^2 - 12x + 34 = 0$$

$$x^2 - 12x + 36 = -34 + 36$$

$$(x-6)^2 = 2$$

$$\sqrt{(x-6)^2} = \sqrt{2}$$

$$x-6 = \pm\sqrt{2}$$

Ans:  $x = 6 \pm \sqrt{2}$

$$x^2 + 6x - 12 = 0$$

$$x^2 + 6x + 9 = 12 + 9$$

$$\sqrt{(x+3)^2} = \sqrt{21}$$

$$x+3 = \pm\sqrt{21}$$

Ans  $x = -3 \pm \sqrt{21}$

$$2x^2 + 12x = -5$$

$$\left(\frac{1}{2}\right)2(x^2 + 6x) = -5\left(\frac{1}{2}\right)$$

Factor out 2

$$x^2 + 6x + 9 = -\frac{5}{2} + 9$$

$$\sqrt{(x+3)^2} = \sqrt{\frac{13}{2}}$$

$$x+3 = \pm\sqrt{\frac{13}{2}}$$

Ans  $x = -3 \pm \sqrt{\frac{13}{2}}$

$$3x^2 - 9x - 30 = 0$$

$$\left(\frac{1}{3}\right)3(x^2 - 3x - 10) = 0$$

$$x^2 - 3x - 10 = 0$$

$$x^2 - 3x + \frac{9}{4} = 10 + \frac{9}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{49}{4}}$$

$$x - \frac{3}{2} = \pm\frac{7}{4}$$

$$x = \frac{3}{2} \pm \frac{7}{4}$$

Using completing the square to rewrite in vertex form:  $y = (x - h)^2 + k$

$$y = x^2 + 6x + 2$$

$$y = (x^2 + 6x + 9) - 9 + 2$$

$$y = (x+3)^2 - 7$$

Separate constant.  
Add & Subtract = 0

so vertex (h, k) = (-3, -7)  
so equation has not been changed

$$y = x^2 - 10x - 2$$

$$y = x^2 - 10x + 25 - 25 - 2$$

$$y = (x-5)^2 - 27$$

$$\frac{1}{2}(-10) = -5 \quad -5^2 = 25$$

$\therefore (h, k) = (5, -27)$

$$y = x^2 + 5x + 3$$

$$y = x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 3$$

$$y = \left(x + \frac{5}{2}\right)^2 - \frac{13}{4}$$

And vertex is  $\left(-\frac{5}{2}, -\frac{13}{4}\right)$

$$\frac{1}{2}(5) = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$-\frac{25}{4} + 3 = -\frac{25}{4} + \frac{12}{4} = -\frac{13}{4}$$