Algebra 2

Lesson Ch4: Matrices – Solving systems of 3 Variables By Hand

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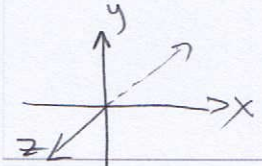
All systems of 3 equations would be really nice to solve if they were all like:
so…. solve

$$\begin{aligned} x - 2y + z &= -4 \\ -4x + y &= -7 \\ z &= -4 \end{aligned}$$



$x - 2y - 4 = -4$ ($+4$ $+4$)

$4(x - 2y) = (0)$

$-4x + y = -7$

$4x - 8y = 0$

$-7y = -7$

$\underline{y = 1}$

$x - 2(1) + (-4) = -4$

$x - 2 - 4 = -4$

$x - 6 = -4$ ($+6$ $+6$)

$x = 2$

x, y, z

Answer $(2, 1, -4)$

Using elimination:

$$\begin{cases} x - 3y + 3z = -4 \\ 2x + 3y - z = 15 \\ 4x - 3y - z = 19 \end{cases}$$

$6x - 2z = 34$

$x - 3\cancel{y} + 3z = -4$

$2x + \cancel{3y} - z = 15$

$3x + 2z = 11$

$3x + 2\cancel{z} = 11$

$6x - \cancel{2z} = 34$

$9x = 45$

$\underline{x = 5}$

$3(5) + 2z = 11$

$15 + 2z = 11$ (-15 -15)

$2z = -4$

$\underline{z = -2}$

$5 - 3(y) + 3(-2) = -4$

$5 - 3y - 6 = -4 + 6 - 5$ (-5 $+6$)

$-3y = -3$

$\underline{y = 1}$

$\boxed{\text{Answer } (5, 1, -2)}$

and….

$$\begin{cases} 2x + y - z = 5 \\ 3x - y + 2z = -1 \\ x - y - z = 0 \end{cases}$$

$x = 1 \quad z = -1$

$1 - y + 1 = 0$

$-y + 2 = 0$

$-y = -2$

$\underline{y = 2}$

$2x + \cancel{y} - z = 5$

$3x - \cancel{y} + 2z = -1$

$(2)(5x + z) = (4)2$

$10x + \cancel{2z} = 8$

$3x - \cancel{2z} = 5$

$13x = 13$

$\underline{x = 1}$

$2x + \cancel{y} - z = 5$

$x - \cancel{y} - z = 0$

$3x - 2z = 5$

$5x + z = 4$

$5(1) + z = 4$

$5 + z = 4$

$\underline{z = -1}$

Answer (x, y, z)

$\underline{(1, 2, -1)}$

Gaussian Elimination

Gaussian Elimination (aka row echelon form) is an effective algorithm (a step by step procedure for calculations) that may be used to reduce systems of 3 equations into a triangular shaped form:

$$\begin{aligned} x - 2y + z &= -4 \\ -4x + y &= -7 \\ z &= -4 \end{aligned}$$

In a college level algebra class you would learn how to perform Gaussian elimination to a matrix, in this class we will work with systems of equations. We saw last class that systems are easily converted to matrix equations.

To perform Gaussian Elimination on a system of equations, one uses a sequence of elementary row operations to modify the system until the last row of the system is a variable equal to a number, the second row is 2 variables equal to a number and the 1st row is 3 variables equal to a number.

There are three types of elementary row operations:

1) Swapping two rows,
2) Multiplying a row by a non-zero number, and
3) Adding a multiple of one row to another row.

NOTE: You will <u>use one row to change another</u> without actually changing the one row. For example: Below we will add – 4 times row 3 to row 2 so to change row 2.

Let's put the words to a problem:

Solve using the Gaussian Method:

$$\begin{cases} 2x - y + 3z = 13 \\ 4x + 3y - 2z = 5 \\ (-4)\ x - y - 4z = -4\ (-4) \end{cases} \xrightarrow{-4R_3 + R_2} \begin{aligned} 2x - y + 3z &= 13 \\ y + 2z &= 3 \\ (-2)\ x - y - 4z &= -4\ (-2) \end{aligned}$$

$$\xrightarrow{-2R_3 + R_1} \begin{aligned} (-1)\ y + 11z &= 21\ (-1) \\ y + 2z &= 3 \\ x - y - 4z &= -4 \end{aligned}$$

$$\xrightarrow{-1R_1 + R_2} \begin{aligned} y + 11z &= 21 \\ z &= 2 \\ x - y - 4z &= -4 \end{aligned}$$

Rearrange. $\begin{aligned} x - y - 4z &= -4 \\ y + 11z &= 21 \\ \underline{\underline{z}} &\underline{\underline{= 2}} \end{aligned}$

$y + 11(2) = 21$

$y + 22 = 21$ $\quad -22 \quad -22$

$\underline{\underline{y = -1}}$

$x + 1 - 4(2) = -4$

$x + 1 - 8 = -4$

$x - 7 = -4$ $\quad +7 \quad +7$

$\underline{\underline{x = 3}}$

Answer

$\underline{\underline{(3, -1, 2)}}$

Work:

$-4\cancel{x} + 4y + 16z = 16$

$\cancel{4x} + 3y - 2z = 5$

$R_2 \quad \frac{1}{7}(7y + 14z) = 21\left(\frac{1}{7}\right)$

$y + 2z = 3$

factor out 7 by multiplying by $\frac{1}{7}$

$-2\cancel{x} + 2y + 8z = 8$

$\cancel{2x} - y + 3z = 13$

$y + 11z = 21$

$-\cancel{y} - 11z = -21$

$\cancel{y} + 2z = 3$

$-9z = -18$

$\underline{\underline{z = 2}}$

$$\begin{cases} x - 2y + 3z = 4 \\ 2x + y - 4z = 3 \\ -3x + 4y - z = -2 \end{cases}$$

(-2) (-2) (3) (3)

$\xrightarrow{-2R_1+R_2}$ $x - 2y + 3z = 4$
$y - 2z = -1$
$-3x + 4y - z = -2$

$-\cancel{2x} + 4y - 6z = -8$
$\cancel{2x} + y - 4z = 3$
$\frac{1}{5}(5y - 10z) = -5(\frac{1}{5})$
R_2 $y - 2z = -1$

$\xrightarrow{3R_1 \to R_3}$ $x - 2y + 3z = 4$
$y - 2z = -1$
$-y + 4z = 5$

$3\cancel{x} - 6y + 9z = 12$
$-\cancel{3x} + 4y - z = -2$
$(\frac{1}{2})\ -2y + 8z = 10\ \frac{1}{2}$
$-y + 4z = 5$

$R_2 + R_3$ $x + 2y + 3z = 4$
$y - 2z = -1$
$z = 2$

$y - 2z = -1$
$-y + 4z = 5$
$2z = 4$
$z = 2$

$y - 2(2) = -1$
$y - 4 = -1 + 4$
$y = 3$

$x - 2(\cancel{3}) + 3(\cancel{2}) = 4$
$x = 4$

$\boxed{(4, 3, 2)}$