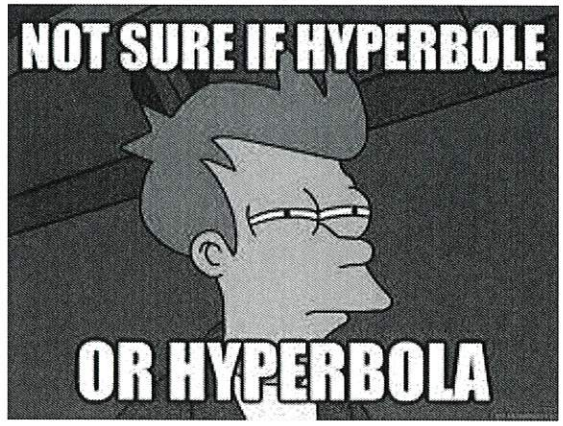
Precalculus
Lesson 10.4: The Hyperbola
Mrs. Snow, Instructor

I will: *be able to graph a hyperbola with the vertex at the origin and solve real work examples involving hyperbolas.*

We will: *Analyze hyperbolas with the center at the origin and solve application problems involving hyperbolas.*



A **hyperbola** is the collection (locus) of all points In the plane, the difference of whose distances from two fixed points, called the foci, is a constant.

Equation of a Hyperbola Centered about the origin with Transverse Axis along the x-axis

Transverse axis : variable is first and always positive
(major axis)

Who is first? Who is positive

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where

$$b^2 = c^2 - a^2$$

relationship between major/minor axes & focus

"a" is always the denominator in the first term. No matter value of the number.

center at $(0,0)$; foci at $(\pm c, 0)$; and vertices at $(\pm a, 0)$

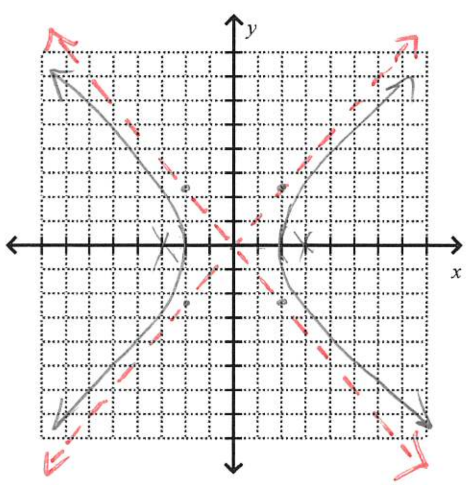
Slope $= \frac{\text{rise}}{\text{run}} = \frac{y \text{ denom.}}{x \text{ denom.}}$

two oblique asymptotes: $y = \pm\frac{b}{a}x$

Find an equation of the hyperbola with center at the origin, one focus at $(3, 0)$ and one vertex at $(-2, 0)$. Graph

foci $(\pm 3, 0)$
vertices $(\pm 2, 0)$

on x-axis



$c = 3$ = distance from origin to focus

$a = 2$ = distance from origin to vertex

$b^2 = c^2 - a^2 \rightarrow b^2 = 9 - 4 = 5$

$b = \pm\sqrt{5}$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Asymptote Slope:

$\frac{\text{rise}}{\text{run}} = \frac{b}{a}$

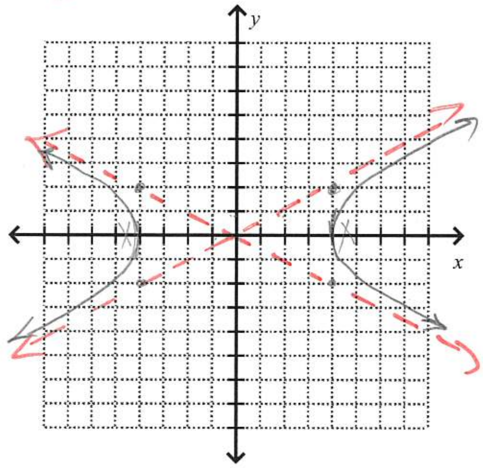
$= \frac{\pm\sqrt{5}}{2}$

Note: guide points for asymptote are in line with the vertex

Analyze the equation; find the center, transverse axis, vertices, and foci. Graph.

$$\frac{\overset{a^2}{x^2}}{16} - \frac{y^2}{\underset{b^2}{4}} = 1$$

Horizontal



$b^2 = c^2 - a^2$

$4 = c^2 - 16$

$20 = c^2$

$\pm\sqrt{20}$ ∴ between 4 & 5

Asymptote slope

$m = \frac{\pm 2}{4} = \pm\frac{1}{2}$

- transverse axis = x
- center (0,0)
- vertices (±4, 0)
- foci $(\pm\sqrt{20}, 0) = (\pm 2\sqrt{5}, 0)$

Equation of a Hyperbola; Center at (0, 0); Transverse Axis along the y-axis

vertical orientation

y is first & positive

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

"a" is with the first term & ∴ positive

$$b^2 = c^2 - a^2$$

center at $(0, 0)$; foci at $(0, \pm c)$; and vertices at $(0, \pm a)$

$$\textit{two oblique asymptotes: } y = \pm\frac{a}{b}x$$

Slope $\frac{\text{rise}}{\text{run}}$

$\pm\frac{\Delta y}{\Delta x}$

note now $\frac{a}{b} = m$

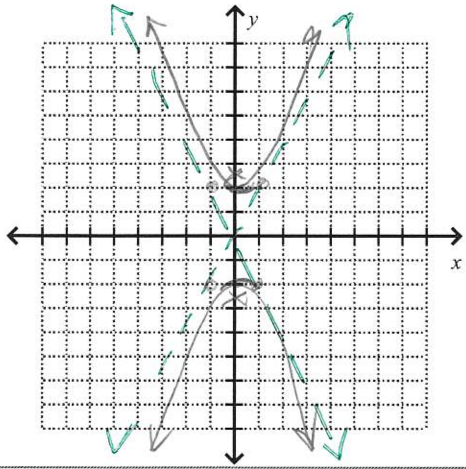
$\frac{y \text{ denominator}}{x \text{ denominator}}$

Analyze the equation, find the center, transverse axis, vertices, and foci and graph:

(#1) get into correct form

$$\frac{y^2}{4} - \frac{4x^2}{4} = \frac{4}{4} \Rightarrow \frac{y^2}{4} - \frac{x^2}{1} = 1$$

a^2 → 4, 1 ← b^2



$b^2 = c^2 - a^2$

$1 = c^2 - 4$

$5 = c^2$

$\pm\sqrt{5} = c$

- center $(0,0)$

- Transverse axis $= y$

Vertices $= (0, \pm 2)$

foci $(0, \pm\sqrt{5})$

Asymptote slope

$\frac{y}{x} = \frac{2}{1} \pm$

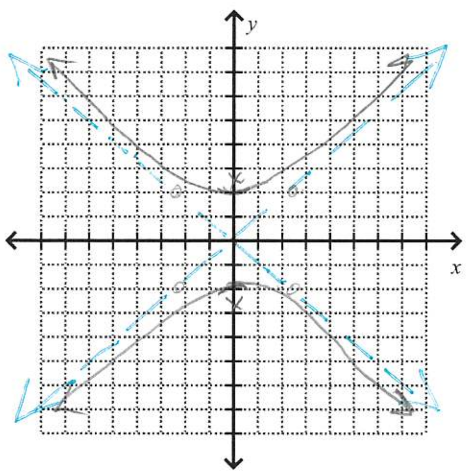
Find an equation of the hyperbola having one vertex at $(0,2)$ and foci at $(0,-3)$ and $(0,3)$. Graph.

Vertical:

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$a = 4$

$c = 3$



$b^2 = c^2 - a^2$

$b^2 = 9 - 4 = 5$

Asymptote slope $= \frac{y}{x}$

$\pm \frac{2}{\sqrt{5}}$

between 2 & 3

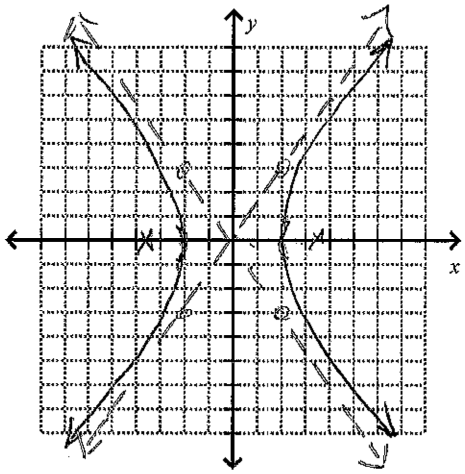
$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

Analyze the equation, find the center, transverse axis , vertices, foci, and asymptotes and graph:

$$\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$$

$$\boxed{\frac{x^2}{4} - \frac{y^2}{9} = 1}$$

hyperbola equation always = 1 ∴ divide by 36



Horizontal axis

Center $(0,0)$

Vertices $(\pm 2, 0)$

Foci $(\pm\sqrt{13}, 0)$

↑ between 3 & 4

$b^2 = c^2 - a^2$

$9 = c^2 - 4 \quad c^2 = 13$

$m = \frac{3}{2} = \frac{y}{x}$

**The homework may ask for the equation of the asymptote. For the quiz and test, <u>all you will be expected to answer is the slope of the asymptote line.</u>*