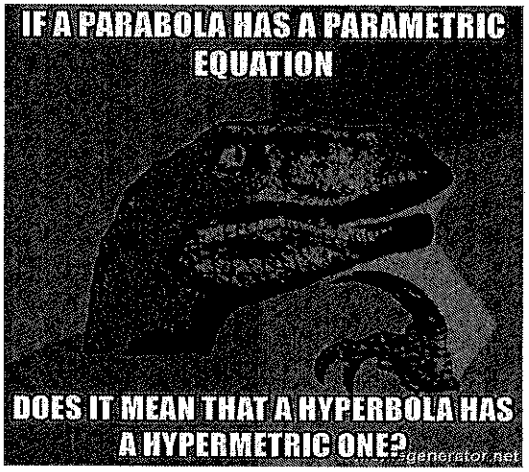
Precalculus
Lesson 10.7: Plane Curves and Parametric Equations
Mrs. Snow, Instructor

I will: *be able to graph parametric equations by hand. I will be able to show how to convert a x-y equation into parametric form and a parametric equation into x-y form. I will be able to talk about how time relates to the x-y values on a graph.*

We will: *graph parametric equations by hand and with the use of a graphing calculator. We will be able to find an equation involving x and y coordinates for a curve that is written in parametric form and find the parametric form an x-y equation. We will see how time can be used as a parameter in parametric equations*



Think of a point moving in a plane through time. The x- and y- coordinates of the point will then be a function of time. So:

Let $x = f(t)\ and\ y = g(t)$ where $f\ and\ g$ are two functions whose common domain is some interval, I. The collection of points defined by

$$(x, y) = (f(t), g(t))$$

is called a **plane curve.** The equations

$$x = f(t) \qquad y = g(t)$$

where t is in I are **parametric equations** for the curve. the variable t is called **parameter.**

Graphing a Curve Defined by Parametric Equations: Notice that for every value of t, we get a point on the curve.

$x = 3t^2 \quad y = 2t$

$-2 \le t \le 2$

t	x	y
-2	12	-4
-1	3	-2
0	0	0
1	3	2
2	12	4

Now find the rectangular equation for the parametric curve. And stated the domain.

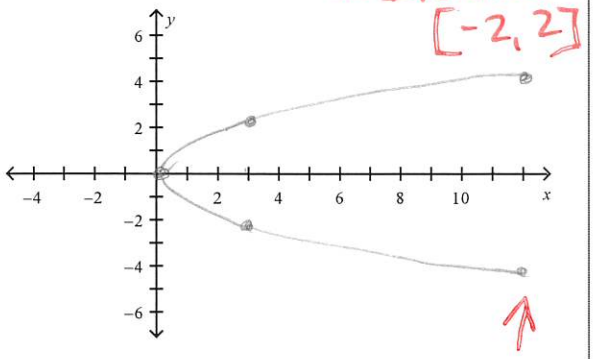
$x^2 = 3t^2 \qquad y = 2t$

$\frac{1}{2}y = t$ substitute

$x^2 = 3\left(\frac{1}{2}y\right)^2$

$x^2 = \frac{3y^2}{4}$

$y^2 = \frac{4}{3}x$



*Interval [-2, 2]

*no arrows as there are min & max values for t

Eliminating the Parameter: (just did this in the previous example!)
Often a curve given by parametric equations can also be represented by a single rectangular equation in x and y. The process of finding this equation is called eliminating the parameter.

Find the rectangular equation for the plane curve defined by the parametric equations. Determine the domain of x.

$x = 4t, \quad y = t - 3 \quad -2 \le t \le 2$

→ solve for "t"

$\frac{1}{4}x = t$ substitute into "y" equation

$y = t - 3$

$y = \frac{1}{4}x - 3$

x-values are determined by the t-values:

find x at $t = -2$

$t = 2$

$x = 4(-2) = -8$

$x = 4(2) = 8$

Domain $[-8, 8]$

$-8 \le x \le 8$

Find the rectangular equation of the curve whose parametric equations are:

$x = 4\cos t,\ and\ y = 3\sin t \qquad -0 \le t \le 2\pi$

① Solve for sine & cosine and square equations

② What trig identity involves both sine & cosine?

③ Substitute

④ From 10.3 we see we have an eclipse.

$\cos t = \frac{x}{4} \rightarrow \cos^2 t = \frac{x^2}{16}$

$\sin t = \frac{y}{3} \rightarrow \sin^2 t = \frac{y^2}{9}$

$\sin^2 t + \cos^2 t = 1$

$\frac{y^2}{9} + \frac{x^2}{16} = 1$

OR $\frac{x^2}{16} + \frac{y^2}{9} = 1$