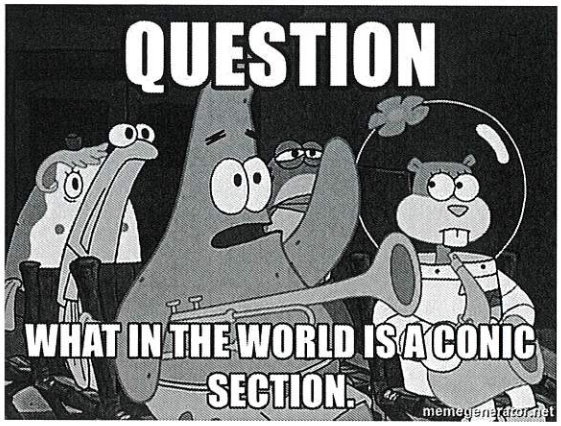
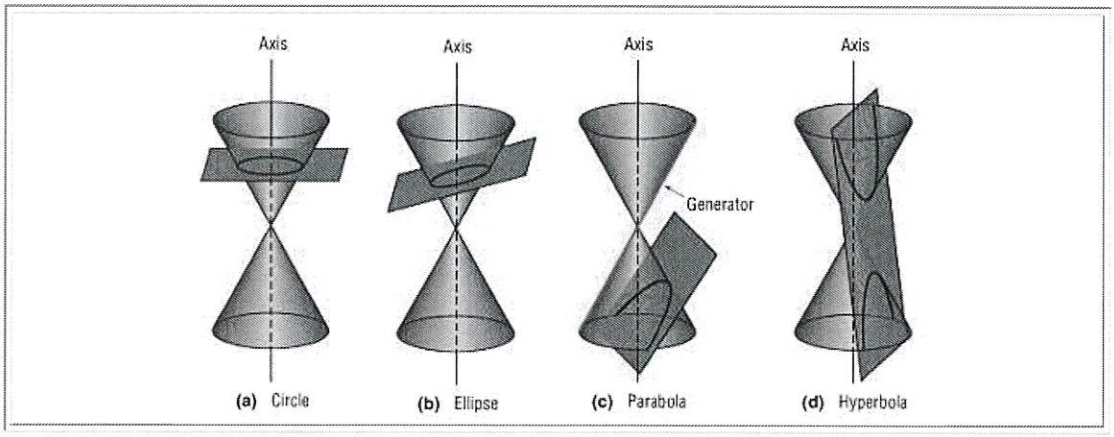
Precalculus
Lesson 10.2: The Parabola
Mrs. Snow, Instructor

I will *be able to graph a parabola with the vertex at the origin and solve real world examples involving parabolas*

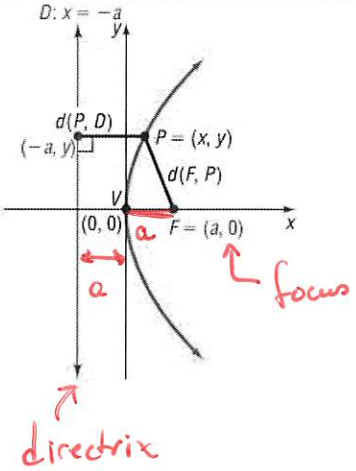
We will: *Analyze parabolas with a vertex at the origin and solve application problems involving parabolas*



Conic sections are curves that result from the intersection of a cone and a plane. We will be looking at all four curves: circle, parabola, ellipse and the hyperbola.



Parabola: A collection, or locus, of all points P in the plane that are the same distance from a fixed point as they are from a fixed line. The point F is the **focus** and the line is its **directrix.**



these distances are equal:

$$d(F, P) = d(P, D)$$

For the parabola that opens along the x-axis:

$$y^2 = 4ax$$

where:

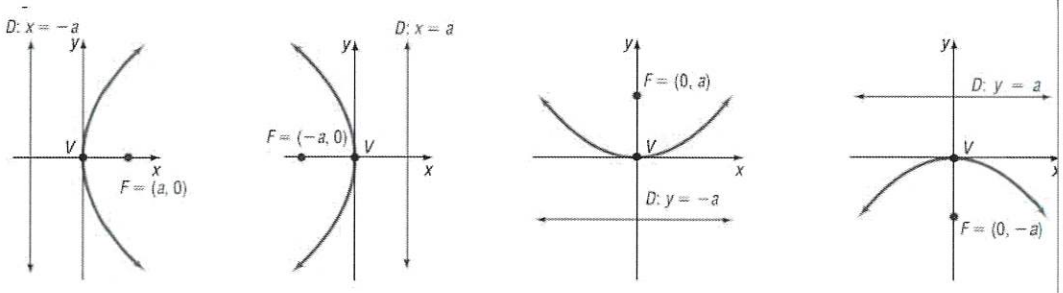
$vertex\ at\ (0, 0)$, $focus\ at\ (a, 0)$,

"a "is the distance from the vertex to the focus of a parabola and distance from the vertex to the directrix

A parabola will open onto the positive or negative x- or y-axes:

Equations of a Parabola, Vertex at (0, 0) and the Focus is on an Axis

vertex	focus	directrix	equation	description
$(0,0)$ $(0,0)$	$(a,0)$ $(-a,0)$	$x=-a$ $x=a$	$y^2=4ax$ $y^2=-4ax$	opens on the positive x-axis opens on the negative x-axis
$(0,0)$ $(0,0)$	$(0,a)$ $(0,-a)$	$y=-a$ $y=a$	$x^2=4ay$ $x^2=-4ay$	opens on the positive y-axis opens on the negative y-axis



(a) $y^2 = 4ax$ (b) $y^2 = -4ax$ (c) $x^2 = 4ay$ (d) $x^2 = -4ay$

Analyze the Equation of a Parabola

Analyze the equation: $y^2 = 8x$

positive ↑ x axis

analyze??? (find the vertex, focus and directrix and graph)

$y^2 = 8x \quad = 4ax$

$8 = 4a$

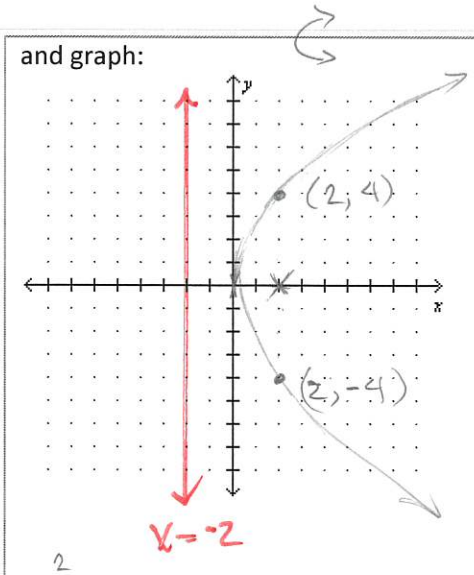
$2 = a$

find 2 points

vertex $(0,0)$
focus $(2,0)$
directrix $x = -2$

2 points: use the x-element here, 2, to find points on the parabola.

and graph:



$y^2 = 8(2)$

$y^2 = 16$

$y = \pm 4$

Opens on negative y-axis

Analyze the equation: $x^2 = -12y$ (find the vertex, focus and directrix and graph)

$x^2 = -12y = 4ay$

$-3 = a$

vertex = (0,0)

focus = (0, -3)

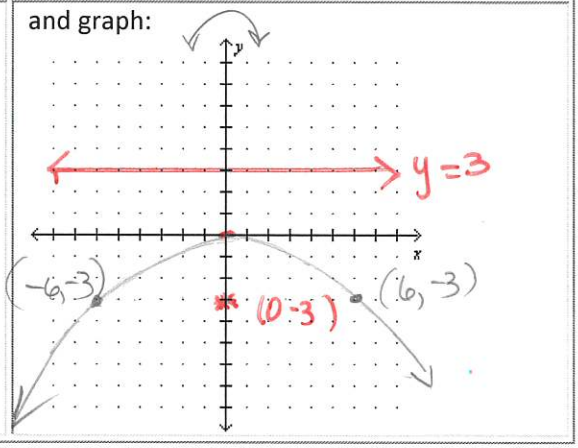
directrix → $y = 3$

2 points at $y = -3$

$x^2 = -12(-3)$

$x = \pm 6$

and graph:



Graphing and Finding Equations of Parabolas

Find an equation of a parabola with a vertex at (0,0) and a focus at (3,0). Graph the equation

focus (3, 0) → a = 3

$y^2 = 4ax$

$y^2 = 4(3)x \Rightarrow \boxed{y^2 = 12x}$

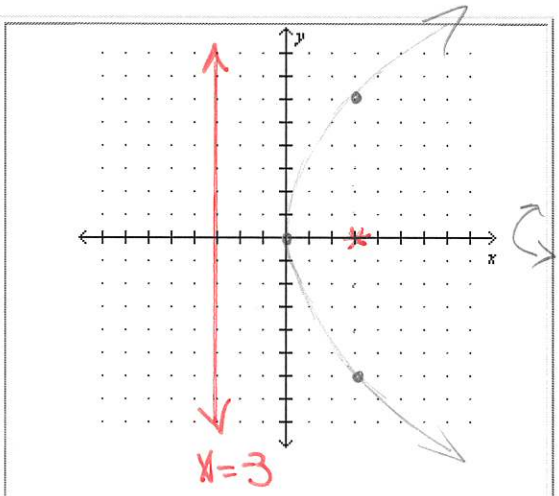
directrix $x = -3$

2 points at $x = 3$

$y^2 = 12(3)$

$y^2 = 36$

$y = \pm 6$



Find an equation of a parabola with a focus at (0,4) and a and directrix line $y = -4$ Graph the equation

$x^2 = 4ay$

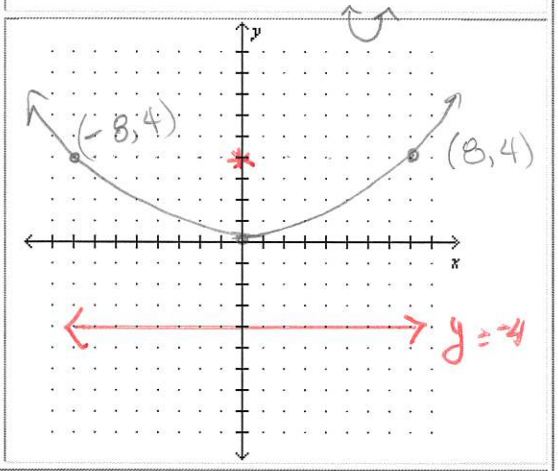
$x^2 = 4(4)y \Rightarrow \boxed{x^2 = 16y}$

2 points at $y = 4$

$x^2 = 16(4)$

$x^2 = 64$

$x = \pm 8$



Find the equation of the parabola with vertex at (0, 0) if its axis of symmetry is the x-axis and its graph contain the point $(-\frac{1}{2}, 2)$

Draw: $(-\frac{1}{2}, 2)$ in quadrant II

Well

$y^2 = 4ax$ plug in $(-\frac{1}{2}, 2)$

$2^2 = 4(a)(-\frac{1}{2})$

$4 = -2a$

$\boxed{-2 = a}$

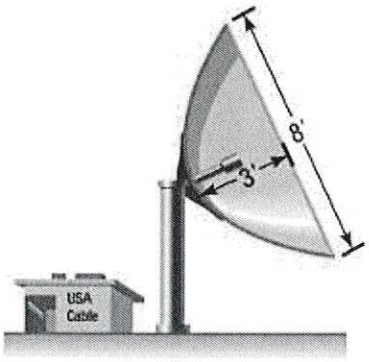
$\boxed{y^2 = -8x}$

A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 8 feet across at its opening and 3 feet deep at its center, at what position should the receiver be placed? That is, where is the focus?

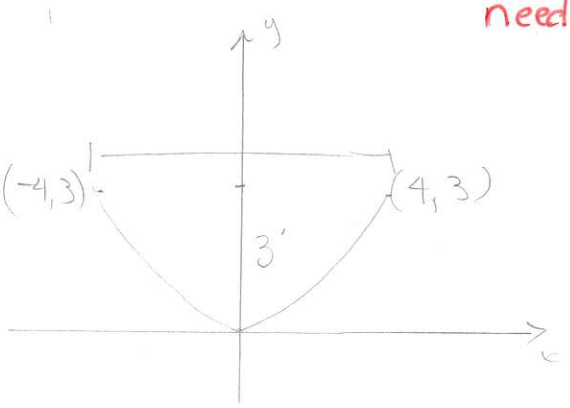
focus

Sketch

need "a"



(a)



$x^2 = 4ay$ Use point

focus $(0, a)$

point on graph $= (4, 3)$

$4^2 = 4(a)(3)$

$16 = 12a$

$\frac{16}{12} = a$

$\boxed{\frac{4}{3} = a}$

Ans Receiver $1\frac{1}{3}$ feet from the base of the dish - that is 1 foot, 4 inches.