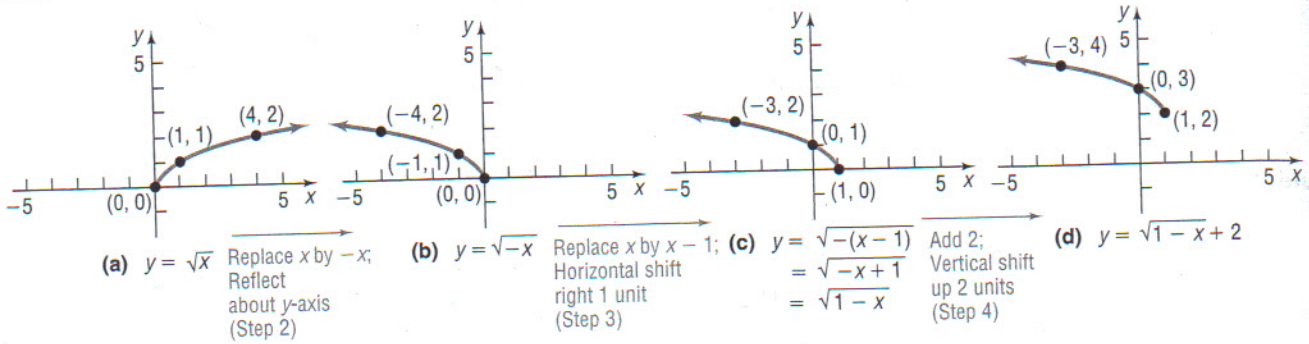
See Figure 58.

Figure 58



The domain of f is $(-\infty, 1]$ and its range is $[2, \infty)$.

Now Work PROBLEM 61

2.5 Assess Your Understanding

1, 2, 3, 4, 5, 7, 11, 15, 17, 19, 21, 23, 27, 29, 69, 71

Concepts and Vocabulary

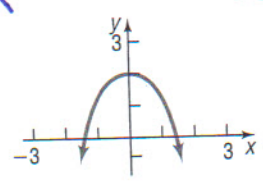
1. Suppose that the graph of a function f is known. Then the graph of $y = f(x - 2)$ may be obtained by a(n) _____ shift of the graph of f to the _____ a distance of 2 units.
2. Suppose that the graph of a function f is known. Then the graph of $y = f(-x)$ may be obtained by a reflection about the _____-axis of the graph of the function $y = f(x)$.
3. Suppose that the graph of a function g is known. The graph of $y = g(x) + 2$ may be obtained by a _____ shift of the graph of g _____ a distance of 2 units.
4. ***True or False*** The graph of $y = -f(x)$ is the reflection about the x-axis of the graph of $y = f(x)$.
5. ***True or False*** To obtain the graph of $f(x) = \sqrt{x + 2}$, shift the graph of $y = \sqrt{x}$ horizontally to the right 2 units.
6. ***True or False*** To obtain the graph of $f(x) = x^3 + 5$, shift the graph of $y = x^3$ vertically up 5 units. True

Skill Building

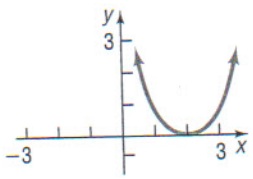
In Problems 7–18, match each graph to one of the following functions:

A. $y = x^2 + 2$
B. $y = -x^2 + 2$
C. $y = |x| + 2$
D. $y = -|x| + 2$
E. $y = (x - 2)^2$
F. $y = -(x + 2)^2$
G. $y = |x - 2|$
H. $y = -|x + 2|$
I. $y = 2x^2$
J. $y = -2x^2$
K. $y = 2|x|$
L. $y = -2|x|$

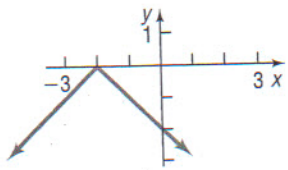
7.



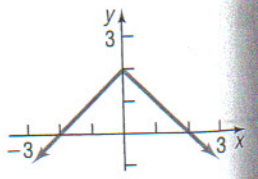
8. E



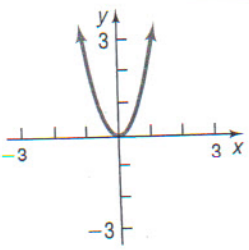
9. H



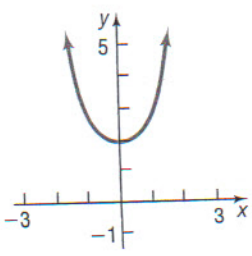
10. D



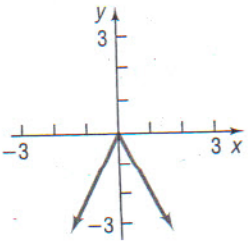
11.



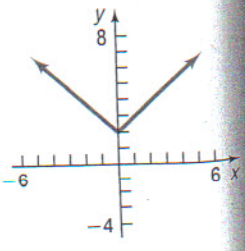
12. A



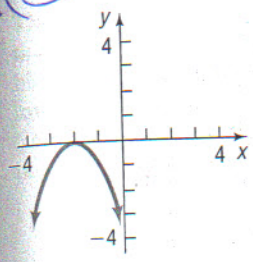
13. L



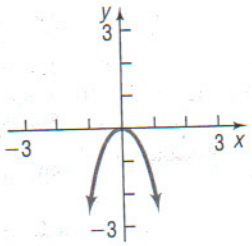
14. C



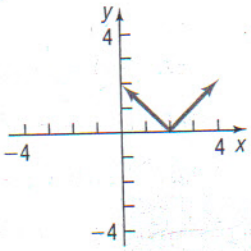
15.



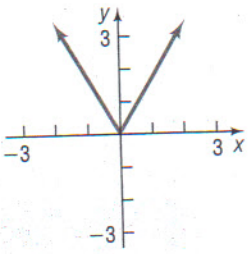
16. J



17.



18. K



In Problems 19–26, write the function whose graph is the graph of $y = x^3$, but is:

19. Shifted to the right 4 units

20. Shifted to the left 4 units $y = (x + 4)^3$

21. Shifted up 4 units

22. Shifted down 4 units $y = x^3 - 4$

23. Reflected about the y-axis

24. Reflected about the x-axis $y = -x^3$

25. Vertically stretched by a factor of 4 $y = 4x^3$

***26.** Horizontally stretched by a factor of 4

In Problems 27–30, find the function that is finally graphed after each of the following transformations is applied to the graph of $y = \sqrt{x}$ in the order stated.

27. (1) Shift up 2 units
(2) Reflect about the x-axis
(3) Reflect about the y-axis)

28. (1) Reflect about the x-axis
(2) Shift right 3 units
(3) Shift down 2 units $y = -\sqrt{x - 3} - 2$

29. (1) Reflect about the x-axis
(2) Shift up 2 units
(3) Shift left 3 units

30. (1) Shift up 2 units
(2) Reflect about the y-axis
(3) Shift left 3 units $y = \sqrt{-(x + 3)} + 2 = \sqrt{-x - 3} + 2$

31. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = -f(x)$? (c)
(a) $(6, 3)$ (b) $(6, -3)$
(c) $(3, -6)$ (d) $(-3, 6)$

32. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(-x)$? (d)
(a) $(6, 3)$ (b) $(6, -3)$
(c) $(3, -6)$ (d) $(-3, 6)$

33. If $(1, 3)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = 2f(x)$? (c)
(a) $\left(1, \frac{3}{2}\right)$ (b) $(2, 3)$
(c) $(1, 6)$ (d) $\left(\frac{1}{2}, 3\right)$

34. If $(4, 2)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(2x)$? (c)
(a) $(4, 1)$ (b) $(8, 2)$
(c) $(2, 2)$ (d) $(4, 4)$

35. Suppose that the x-intercepts of the graph of $y = f(x)$ are -5 and 3.
*(a) What are the x-intercepts of the graph of $y = f(x + 2)$?
*(b) What are the x-intercepts of the graph of $y = f(x - 2)$?
*(c) What are the x-intercepts of the graph of $y = 4f(x)$?
*(d) What are the x-intercepts of the graph of $y = f(-x)$?

36. Suppose that the x-intercepts of the graph of $y = f(x)$ are -8 and 1.
*(a) What are the x-intercepts of the graph of $y = f(x + 4)$?
*(b) What are the x-intercepts of the graph of $y = f(x - 3)$?
*(c) What are the x-intercepts of the graph of $y = 2f(x)$?
*(d) What are the x-intercepts of the graph of $y = f(-x)$?

37. Suppose that the function $y = f(x)$ is increasing on the interval $(-1, 5)$.
(a) Over what interval is the graph of $y = f(x + 2)$ increasing? $(-3, 3)$
(b) Over what interval is the graph of $y = f(x - 5)$ increasing? $(4, 10)$
*(c) What can be said about the graph of $y = -f(x)$?
*(d) What can be said about the graph of $y = f(-x)$?

38. Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.
(a) Over what interval is the graph of $y = f(x + 2)$ decreasing? $(-4, 5)$
(b) Over what interval is the graph of $y = f(x - 5)$ decreasing? $(3, 12)$
*(c) What can be said about the graph of $y = -f(x)$?
*(d) What can be said about the graph of $y = f(-x)$?

In Problems 39–68, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function (for example, $y = x^2$) and show all stages. Be sure to show at least three key points. Find the domain and the range of each function. Verify your results using a graphing utility.

***39.** $f(x) = x^2 - 1$

***40.** $f(x) = x^2 + 4$

***41.** $g(x) = x^3 + 1$

***42.** $g(x) = x^3 - 1$

***43.** $h(x) = \sqrt{x - 2}$

***44.** $h(x) = \sqrt{x + 1}$

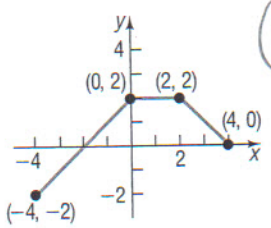
*Due to space restrictions, answers to these exercises may be found in the Answers in the back of the book.

*45. $f(x) = (x - 1)^3 + 2$
*46. $f(x) = (x + 2)^3 - 3$
*47. $g(x) = 4\sqrt{x}$
*48. $g(x) = \frac{1}{2}\sqrt{x}$
*49. $h(x) = \frac{1}{2x}$
*50. $h(x) = \sqrt[3]{2x}$
*51. $f(x) = -\sqrt[3]{x}$
*52. $f(x) = -\sqrt{x}$
*53. $g(x) = \sqrt[3]{-x}$
*54. $g(x) = \frac{1}{-x}$
*55. $h(x) = -x^3 + 2$
*56. $h(x) = \frac{1}{-x} + 2$
*57. $f(x) = 2(x + 1)^2 - 3$
*58. $f(x) = 3(x - 2)^2 + 1$
*59. $g(x) = 2\sqrt{x - 2} + 1$
*60. $g(x) = 3|x + 1| - 3$
*61. $h(x) = \sqrt{-x} - 2$
*62. $h(x) = \frac{4}{x} + 2$
*63. $f(x) = -(x + 1)^3 - 1$
*64. $f(x) = -4\sqrt{x - 1}$
*65. $g(x) = 2|1 - x|$
*66. $g(x) = 4\sqrt{2 - x}$
*67. $h(x) = 2 \text{ int } (x - 1)$
*68. $h(x) = \text{int}(-x)$

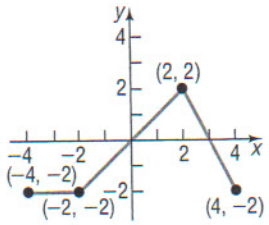
In Problems 69–72, the graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions:

(a) $F(x) = f(x) + 3$ (b) $G(x) = f(x + 2)$ (c) $P(x) = -f(x)$ (d) $H(x) = f(x + 1) - 2$
(e) $Q(x) = \frac{1}{2}f(x)$ (f) $g(x) = f(-x)$ (g) $h(x) = f(2x)$

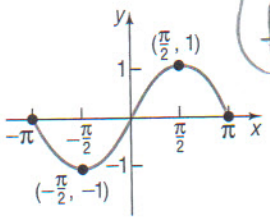
*69.



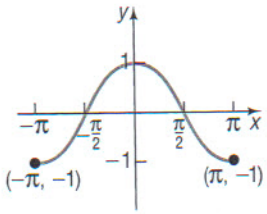
*70.



*71.



*72.



Mixed Practice

73. *(a) Using a graphing utility, graph $f(x) = x^3 - 9x$ for $-4 < x < 4$.
(b) Find the x-intercepts of the graph of f.
*(c) Approximate any local maxima and local minima.
*(d) Determine where f is increasing and where it is decreasing.
*(e) Without using a graphing utility, repeat parts (b)–(d) for $y = f(x + 2)$.
*(f) Without using a graphing utility, repeat parts (b)–(d) for $y = 2f(x)$.
*(g) Without using a graphing utility, repeat parts (b)–(d) for $y = f(-x)$.

74. *(a) Using a graphing utility, graph $f(x) = x^3 - 4x$ for $-3 < x < 3$.
(b) Find the x-intercepts of the graph of f. $-2, 0, 2$
*(c) Approximate any local maxima and local minima.
*(d) Determine where f is increasing and where it is decreasing.
*(e) Without using a graphing utility, repeat parts (b)–(d) for $y = f(x - 4)$.
*(f) Without using a graphing utility, repeat parts (b)–(d) for $y = f(2x)$.
*(g) Without using a graphing utility, repeat parts (b)–(d) for $y = -f(x)$.

In Problems 75–82, complete the square of each quadratic expression. Then graph each function using the technique of shifting. (If necessary, refer to Appendix A, Section A.3 to review completing the square.)

*75. $f(x) = x^2 + 2x$
*76. $f(x) = x^2 - 6x$
*77. $f(x) = x^2 - 8x + 1$
*78. $f(x) = x^2 + 4x + 2$
*79. $f(x) = 2x^2 - 12x + 19$
*80. $f(x) = 3x^2 + 6x + 1$
*81. $f(x) = -3x^2 - 12x - 17$
*82. $f(x) = -2x^2 - 12x - 13$

Applications and Extensions

*83. The equation $y = (x - c)^2$ defines a *family of parabolas*, one parabola for each value of c. On one set of coordinate axes, graph the members of the family for $c = 0$, $c = 3$, and $c = -2$.