Precalculus

Lesson 7.3: Trigonometric Equations

Mrs. Snow, Instructor

We have studied trigonometric graphs and expressions. The next skill we need to learn is how to solve trigonometric equations.

Determine whether or not θ is a solution for the equation below.

$\theta = \frac{\pi}{4}, \quad \theta = \frac{\pi}{6}$ Substitute

$2\sin\theta - 1 = 0$

$2\sin\frac{\pi}{4} - 1 \stackrel{?}{=} 0$

$2\left(\frac{\sqrt{2}}{2}\right) - 1 = 0$

$\sqrt{2} - 1 \neq 0$ NO

$2\sin\frac{\pi}{6} - 1 \stackrel{?}{=} 0$

$2\left(\frac{1}{2}\right) - 1 = 0$

$1 - 1 = 0$ yes

Solve for θ.

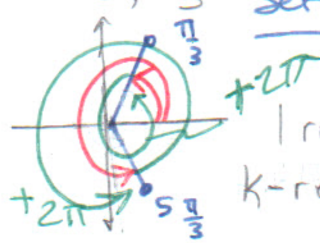
Give a general formula for all the solutions.

List 8 of the solutions.

$\cos\theta = \frac{1}{2}$

for every revolution we get another set of solutions

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$



1 revolution $= 2\pi$

k-revolutions $= 2k\pi$

So for multiples of $\frac{\pi}{3}$ & $\frac{5\pi}{3}$

$\theta = \frac{\pi}{3} + 2k\pi$

$\theta = \frac{5\pi}{3} + 2k\pi$

General solution w/ common denominator: $\theta = \frac{\pi}{3} + \frac{6k\pi}{3}$

$\theta = \frac{5\pi}{3} + \frac{6k\pi}{3}$

8 solutions:

$\frac{\pi}{3}, \frac{7\pi}{3}, \frac{13\pi}{3}, -\frac{5\pi}{3}$ ← clockwise is an option too

$\frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}, -\frac{\pi}{3}$

We can solve linear trigonometric equations:

$$2\sin\theta + \sqrt{3} = 0, \quad 0 \le \theta < 2\pi$$

Use algebraic principles to isolate the variable terms, then trig to solve for θ.

$$\begin{array}{c|c} S & A \\ \hline \star T & C \star \end{array}$$

Sine Negative

$2\sin\theta + \sqrt{3} = 0$

$2\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\boxed{\theta = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$

Sine of what angle is $-\frac{\sqrt{3}}{2}$?

Interval!! $0 \le \theta < 2\pi$

Solve: yikes!

$$\sin(2\theta) = \frac{1}{2}, \quad 0 \le \theta < 2\pi$$

FYI: $\omega = 2$ ∴ horiz. comp.

① Ask where $\sin\theta = \frac{1}{2}$?

$\sin ☁ = \frac{1}{2}$; $☁ = \frac{\pi}{6}, \frac{5\pi}{6}$

So: General form

$☁ = \left(\frac{1}{2}\right)2\theta = \left(\frac{\pi}{6} + 2k\pi\right)\frac{1}{2}$

$\boxed{\theta = \frac{\pi}{12} + k\pi}$

$☁ = \left(\frac{1}{2}\right)2\theta = \left(\frac{5\pi}{6} + 2k\pi\right)\frac{1}{2}$

$\boxed{\theta = \frac{5\pi}{12} + k\pi}$

② Now find all θ in the interval $0 \le \theta < 2\pi$

$k = 0$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$

$k = 1$: $\theta = \frac{\pi}{12} + \pi\left(\frac{12}{12}\right) = \frac{13\pi}{12}$

$\theta = \frac{5\pi}{12} + \pi\left(\frac{12}{12}\right) = \frac{17\pi}{12}$

$k = 2$? No = out of interval.

$\theta = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

Solve: Same process w/ phase shift

$$\tan\left(\theta - \frac{\pi}{2}\right) = 1, \quad 0 \le \theta < 2\pi$$

$\tan ☁ = 1 \qquad ☁ = \frac{\pi}{4}$

Remember that tangent period is: $pd = \frac{\pi}{\omega} = \frac{\pi}{1}$

$☁ = \theta - \frac{\pi}{2} \overset{+\frac{\pi}{2}}{=} \frac{\pi}{4} + k\pi \quad \left(+\frac{\pi}{2}\right)$

$\boxed{\theta = \frac{3\pi}{4} + k\pi}$

$k = 0 \quad \theta = \frac{3\pi}{4}$

$k = 1 \quad \theta = \frac{3\pi}{4} + \pi\frac{4}{4} = \frac{7\pi}{4}$

$k = 2 \quad \theta = \frac{3\pi}{4} + 2\pi$ out of interval

$\theta: \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

Solving a trigonometric quadratic equation:

$$2sin^2\theta - 3sin\theta + 1 = 0 \ , \ 0 \le \theta < 2\pi$$

to solve quadratic equations: factor

let $X = \sin\theta$ ∴
$X^2 = \sin^2\theta$

Finish

Tris Answers!

So: $2X^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$2x - 1 = 0 \quad x - 1 = 0$

$x = \frac{1}{2} \quad x = 1$

$\sin\theta = \frac{1}{2} \quad \sin\theta = 1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{\pi}{2}$

$\theta = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$

Solving with trigonometric identities:

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$3cos\theta + 3 = 2sin^2\theta \ , \ 0 \le \theta < 2\pi$$

$3\cos\theta + 3 = 2(1 - \cos^2\theta)$

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$X = \cos\theta, \ X^2 = \cos^2\theta$

$2x^2 + 3x + 1 = 0$

$(2x+1)(x+1) = 0$

$2x + 1 = 0 \quad x + 1 = 0$

$x = -\frac{1}{2} \quad x = -1$

Tris format:

$\cos\theta = -\frac{1}{2} \quad \cos\theta = -1$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \theta = \pi$

$\theta = \left\{ \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$cos^2\theta + sin\theta = 2 \ , \ 0 \le \theta < 2\pi$$

(-1) $1 - \sin^2\theta + \sin\theta - 2 = 0$ (-1)

$\sin^2\theta - \sin\theta + 1 = 0$

$X = \sin\theta \quad X^2 = \sin^2\theta$

$X^2 - x + 1 = 0$

Yikes!

Cannot factor, are there any real solutions? Check discriminant.

$b^2 - 4ac = 1 - 4(1)(1)$
$= -3$ ← neg.

∴ No real solutions

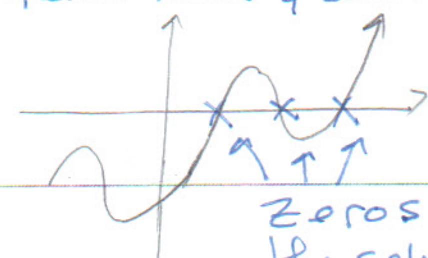
Graphing utilities are always nice…. solve, rounding to two decimal places.

$$5\sin x + x = 3$$

put into "y" form;
everything on side of the equation

$$y = 5\sin x + x - 3$$

* Radian mode & zoom trig



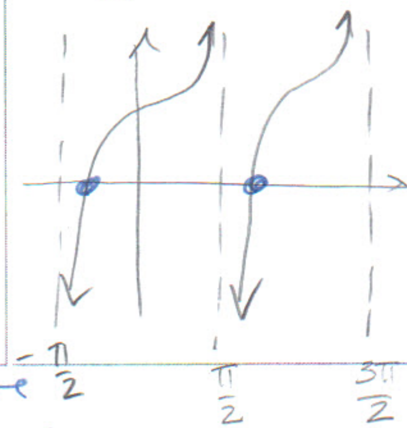
Zeros are the solutions

$$x = \{.52, 3.18, 5.71\}$$

$$tan\theta = -2\ ,\ \ 0 \le \theta < 2\pi$$

$$y = \tan\theta + 2$$

Window
x min: 0
x max: 2π
x scl: $\frac{\pi}{4}$



$$\theta = \{2.03, 5.18\}$$