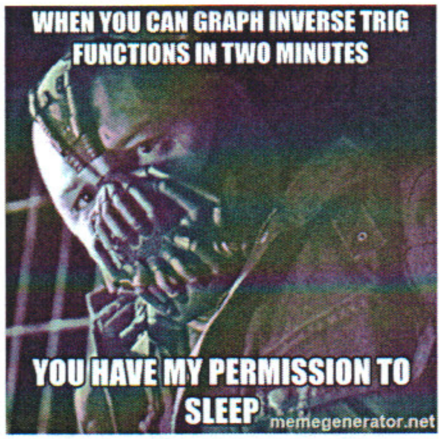
Precalculus

Lesson 7.1: The Inverse Sine, Cosine and Tangent Functions

Mrs. Snow, Instructor

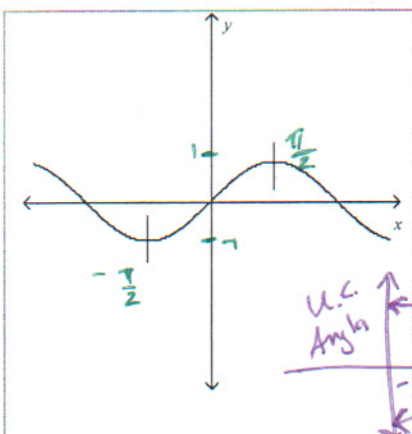
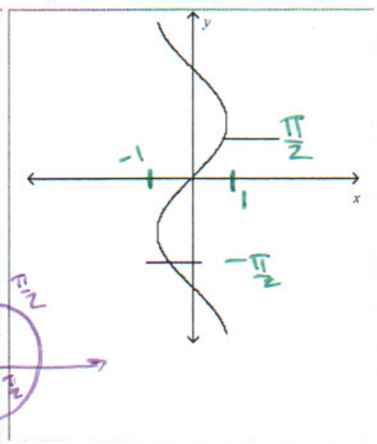
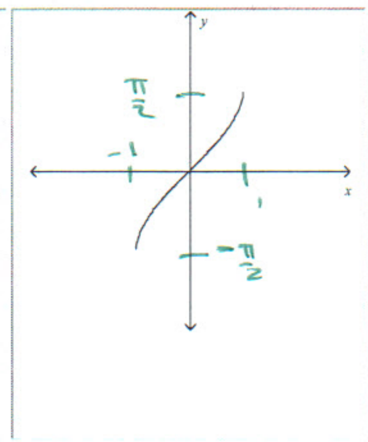


Inverse: A mathematical operation that is the opposite effect of another operation. The operation undoes what the first operation did! Some examples of inverse operations include addition and subtraction and multiplication and division.

In 5.2 we studied inverse functions. If a function is one-to-one it has an inverse function (one function undoes the other). To make a function one-to-one, place restrictions on the domain. While trig functions are of course functions, they are not all one-to-one. By placing limits on the domain, a trig function may be forced into being one-to-one.

Remember to find the inverse simply switch the x and y values; $x\ is\ y$ and $y\ is\ x$. Switch the domain and range.

Inverse Sine Function: $\boldsymbol{sin^{-1}}$ is *also known as* <u>*arcsine*</u> *and written as* <u>*arcsin*</u>

		
$\sin x = y$ Domain: (∞, ∞) (restricted domain): $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range: $[-1, 1]$	$\sin^{-1} x = y$ Restrict Sine's domain to allow for an inverse function	$\sin^{-1} x = y$ Domain: $[-1, 1]$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Remember, that inverses undo one another, so if I take the inverse sine of sine, *OR!* the sine of the inverse sine, the operation is undone!

Finding the exact value of an inverse sine function; we are looking for an angle θ, $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

~~sin~~ $\cancel{sin^{-1}}\ 1 = \theta$ sin

Think: The sine of what ∡ is 1?

~~sin~~ ~~sin~~$^{-1}$ 1 = sin θ

sin θ = 1

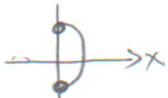
$\boxed{y = \frac{\pi}{2}}$

Watch interval

~~sin~~ $\cancel{sin^{-1}} -\frac{1}{2}$ = sin y

sin y = $-\frac{1}{2}$

y = $-\frac{\pi}{6}$



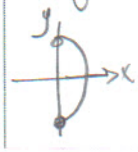
~~sin~~ $\cancel{sin^{-1}}\ \frac{3}{2}$ = sin y

sin y = 3/2

y = ??

$\boxed{\text{Undef.}}$

What angle has an output > 1?



Approximate values of inverse sine functions may be found using a calculator. Express the answer in radians rounded to 2 decimal places. What quadrant will you find these angles?

MODE

$sin^{-1}\frac{1}{3}$ ~.34	$sin^{-1}\left(-\frac{1}{4}\right)$ ~-.25

For composite functions, the domain is dependent upon the domain restrictions of the inner function. This is crucial when determining the value of a composite function where the inner function is outside the domain restrictions.

In the terms of the sine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Find the exact value of composite functions:

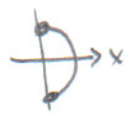
- Can we use composite function rules from above?
- Work from the inside out. ☆
- When sine is the inside function, solve for $sin\ \theta$,
- Then deal with the inverse function, *make sure you pay attention to the domain of x!!!*

$sin^{-1}(\sin\frac{\pi}{4})$

$-\frac{\pi}{2} \le \frac{\pi}{4} \le \frac{\pi}{2}$ True

$\cancel{\sin^{-1}}\left(\cancel{\sin}\frac{\pi}{4}\right) = \frac{\pi}{4}$

$sin^{-1}(\sin\frac{2\pi}{3})$ False



$-\frac{\pi}{2} \le \frac{2\pi}{3} \le \frac{\pi}{2}$

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

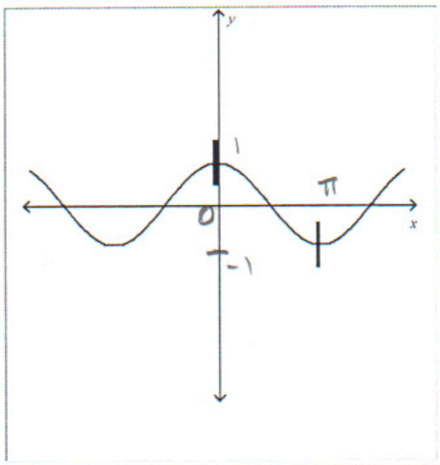
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$

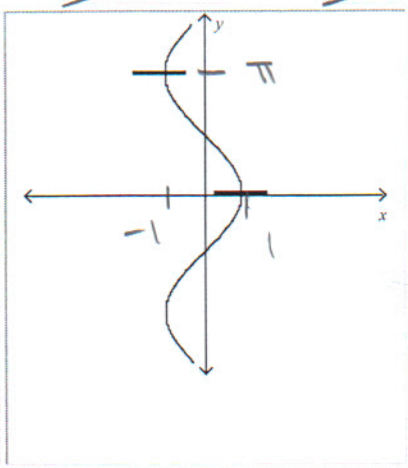
$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \boxed{\theta = \frac{\pi}{3}}$

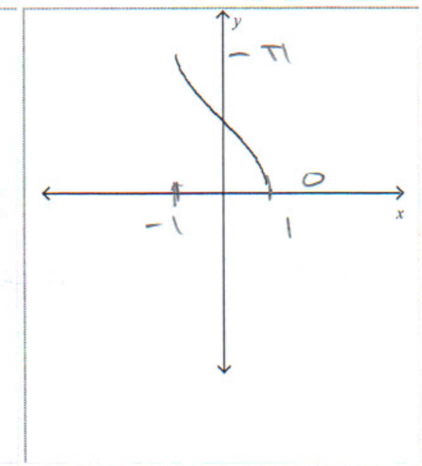
$sin^{-1}(\sin\frac{7\pi}{6})$ not in Domain	$\sin(sin^{-1}(-.5))$	$\sin(sin^{-1}(1.8))$
$\sin\frac{7\pi}{6} = -\frac{1}{2}$	$-1 \le x \le 1$	Sin of what ∡
$\cancel{\sin}\ \sin^{-1}(-\frac{1}{2}) = \theta$ (sin)	True	$= 1.8$
$\sin\theta = \boxed{-\frac{1}{2}}$	$\cancel{\sin}\ \cancel{\sin^{-1}}\ -.5$	Oh, NO Angle!
$\theta = -\frac{\pi}{6}$	$\boxed{= -.5}$	$\boxed{\text{UNDEF}}$

Same process may be used for cosine: arccosine arccos ☺

Inverse Cosine Function: $\boldsymbol{cos^{-1}}$ *also called arrcosine and written as arccsos*







$cos\ x = y$	$\cos^{-1} x = y$	$\cos^{-1} x = y$
Domain: (∞, ∞) (restricted domain): $[0, \pi]$ Range: $[-1, 1]$	Range $[0, \pi]$ Restrict Cosine's domain to allow for an inverse function Domain $[-1, 1]$	Domain: $[-1, 1]$ Range: $[0, \pi]$

Finding the exact value of an inverse cosine function; we are looking for an angle θ, $0 \le \theta \le \pi$

Restriction

$\cancel{\cos}\ \cos^{-1} 0 = \theta$ (cos)	$\cos^{-1} -\frac{\sqrt{2}}{2} = \theta$
cosine of what ∡ $= 0$?	$\frac{3\pi}{4}$
$\theta = \frac{\pi}{2}$	$\cos\theta = -\frac{\sqrt{2}}{2}$
$\cancel{\cos}(\cos^{-1} 0) = \cos\theta$	$\theta = \frac{3\pi}{4}$
$\cos\theta = 0$	
$\theta = \frac{\pi}{2}$	

In the terms of the cosine function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \qquad \text{where } 0 \le x \le \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

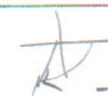
- Can we use composite function rules from above?
- Work from the inside out.
- When cosine is the inside function, solve for $cos\theta$,
- Then deal with the inverse function, *make sure you pay attention to the domain of x!!!*

$\cancel{cos}^{-1}\left(\cancel{cos}\left(\frac{\pi}{12}\right)\right)$

$= \frac{\pi}{12}$

$0 \le \frac{\pi}{12} \le \pi$

$\cancel{cos}^{-1}\,\cancel{cos}\,\frac{2\pi}{3} = cos^{-1}[\cos\left(-\frac{2\pi}{3}\right)]$



$\boxed{= \frac{2\pi}{3}}$ $\cos -\frac{2\pi}{3} = -\frac{1}{2}$

$\cos^{-1} -\frac{1}{2} = \theta$

$\cos\theta = -\frac{1}{2}$

$\theta = 2\pi/3$

Note: cosine is even, so $\cos\left(-\frac{2\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\cos(cos^{-1}\,\pi)$

Interval $-1 \le \pi \le 1$
False

also
cosine of what $\measuredangle = 3.14$?

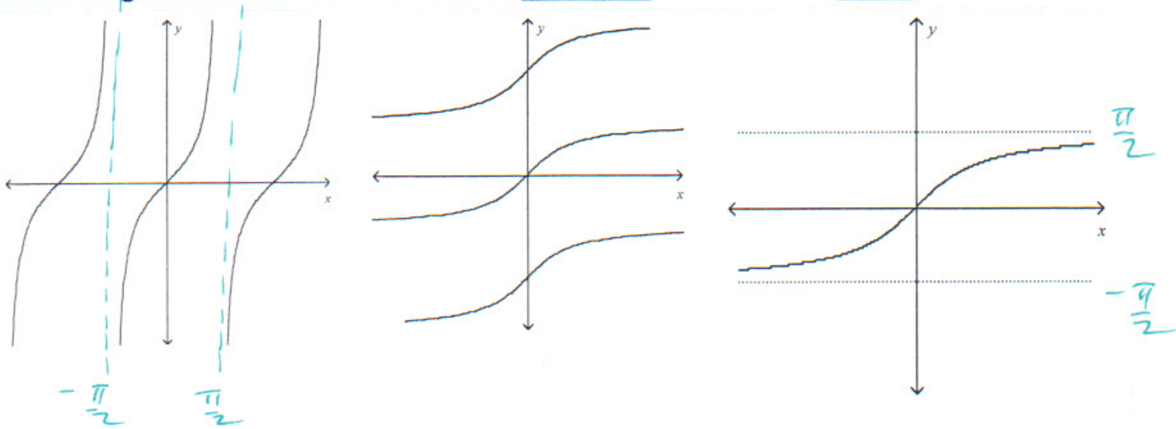
$\boxed{\text{UNDEF.}}$

$\cancel{\cos}\left(\cancel{cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\sqrt{3}}{2}$

$-1 \le \frac{\sqrt{3}}{2} \le 1$

True

Inverse Tangent Function: $\boldsymbol{tan^{-1}}$ *also called* *arctangent* *and written as* *arctan*



$tan\, x = y$

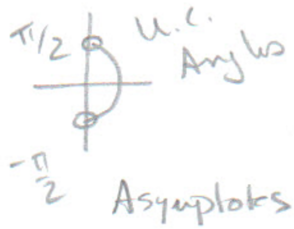
Domain: $(-\infty, \infty)$,

$\neq$ *odd multiples*

Restrict the domain:

$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$



$\tan^{-1} x = y$

Domain: $(-\infty, \infty)$

Range: $= \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

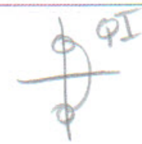
The inverse tangent function is the function tan^{-1} with domain of all real numbers and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ defined by

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$

$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

EXAMPLE Evaluate the inverse tangent functions; find θ for $\frac{-\pi}{2} < \theta < \frac{\pi}{2}\theta$

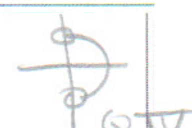
$\tan^{-1} 1$



$\tan \theta = 1$

$\theta = \frac{\pi}{4}$

$\tan^{-1} -\sqrt{3}$



$\tan \theta = -\sqrt{3}$

$\theta = -\frac{\pi}{3}$

$y = \tan^{-1}(-20)$

~ 1.52 (calculator)

In the terms of the tangent function and its inverse, we have the following properties:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where} \quad -\infty < x < \infty$$

Precalculus

Lesson 7.2: The Inverse Trigonometric Functions (continued)

Mrs. Snow, Instructor

Composing a Trig Function

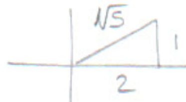
What???? Evaluate a trig function involving inverse functions.

Find the exact value of :

$$\sin\left(tan^{-1}\frac{1}{2}\right) = \sin\theta$$

$\tan^{-1}\frac{1}{2} = \theta$

$\tan\theta = \frac{1}{2} = \frac{O}{A}$



$\sin\theta = \frac{O}{H} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

1. Let θ equal the inverse function
2. By definition: $\theta = tan^{-1}\frac{1}{2}$ $\therefore \tan\theta = \frac{1}{2}$
3. Set up a triangle in which $\tan\theta = \frac{1}{2}$
4. Errors are always made when you do not pay attention to the quadrant the angle is in for inverse functions:

sine: $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

cosine: $0 \le \theta \le \pi$

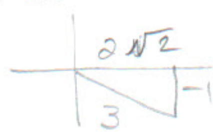
tangent: $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

Rationalize unless instructed differently

$$\cos\left[sin^{-1}\left(-\frac{1}{3}\right)\right] = \cos\theta$$

$\sin^{-1} -\frac{1}{3} = \theta$

$\sin\theta = -\frac{1}{3} = \frac{O}{H}$



$1 + x^2 = 9$

$x^2 = 8$

$x = \sqrt{8}$

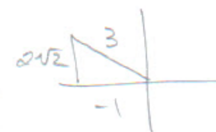
$= 2\sqrt{2}$

$\cos\theta = \frac{A}{H} = \boxed{\frac{2\sqrt{2}}{3}}$

$$\tan\left[cos^{-1}\left(-\frac{1}{3}\right)\right] = \tan\theta$$

$\cos^{-1} -\frac{1}{3} = \theta$

$\cos\theta = -\frac{1}{3} = \frac{A}{H}$



$9 - 1 = 8 = y^2$

$y = 2\sqrt{2}$

$\tan\theta = \frac{O}{A} =$

$= \frac{2\sqrt{2}}{-1}$

$\tan\theta = \boxed{-2\sqrt{2}}$

$$cos^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$$

$\tan -\frac{\pi}{4} = -1$

$\cos^{-1} -1 = \theta$

$\cos\theta = -1$

$\boxed{\theta = \pi}$

Write a Trigonometric expression as an Algebraic Expression:

Look back to the first example. What is the first step?

$$\sin(\tan^{-1} u) = \sin\theta$$

$\tan^{-1} u = \theta$

$\tan\theta = \frac{u}{1} = \frac{O}{A}$ Assume positive

$1^2 + u^2 = h^2$

$\sqrt{1^2+u^2} = h$

$\sin\theta = \frac{O}{H} = \boxed{\frac{u}{\sqrt{1+u^2}}}$

Ok not to rationalize!!