

Chaos Fall Final Exam Review

1. Find the limit:

a.  $\lim_{x \rightarrow 1} (3x^3 - 2x^2 + 4)$

b.  $\lim_{x \rightarrow 1} \frac{x - 3}{x^2 + 4}$

c.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4}$

d.  $\lim_{x \rightarrow \pi} \tan x$

e.  $\lim_{x \rightarrow 0} (2x - 1)^3$

f.  $\lim_{x \rightarrow 4} \frac{x^4 - 4x^3}{x - 4}$

And identify another function that agrees with  $f(x)$  at all but one point.

2. Let  $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \geq 2 \end{cases}$ . Find each limit (if it exists).

a.  $\lim_{x \rightarrow 2^-} f(x)$

b.  $\lim_{x \rightarrow 2^+} f(x)$

c.  $\lim_{x \rightarrow 2} f(x)$

3-4 Find the derivative of the function by using the definition of the derivative.

3.  $f(x) = x^2 - 2x + 3$

4.  $f(x) = \sqrt{x+1}$

5-8 Find the derivative of the function.

5.  $f(x) = 3\cos\theta - \frac{\sin\theta}{4}$

6.  $g(x) = (x^3 - 3x)(x + 2)$

7.  $y = \frac{\sin^2 2x}{x^2}$

8.  $y = 3\cos(3x + 1)$

9-10 Find the second derivative of the function.

9.  $g(t) = t^3 - 3t + 2$

10.

10.  $y = 2x^2 + \sin 2x$

11-12 Find the derivative of the function at the given point.

11.  $f(x) = \sqrt{1 - x^3} \quad (-2, 3)$

12.  $f(x) = \sqrt[3]{x^2 - 1} \quad (3, 2)$

For the following #13-14

- Find the intervals of increase or decrease
- Find the local maximum and minimum values
- Find the intervals of concavity and the inflection points
- Use the information from parts (a-c) to sketch the graph

13.  $f(x) = x^3 + 3x^2 - 9x + 6$

14.  $f(x) = -x^3 + 12x + 5$

For the following #15

- Find the vertical and horizontal asymptotes
- Find the intervals of increase or decrease
- Find the local maximum and minimum values
- Find the intervals of concavity and the inflection points
- Use the information from parts (a-d) to sketch the graph of  $f$ .

15.  $f(x) = \frac{x^2}{(x - 2)^2}$

Locate the absolute extrema of the function on the closed interval.

16.  $f(x) = x^3 - 12x \quad [0, 4]$

17.

$f(x) = \frac{1}{x - 2} \quad [0, 1]$

18. When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/sec. At what rate is the plate's area increasing when the radius is 50cm?
19. A spherical balloon is inflated with helium at the rate of  $100\pi \text{ ft}^3 / \text{min}$ .
- How fast is the balloon's radius increasing at the instant the radius is 5ft?
  - How fast is the surface area increasing at that instant?
20. An open box is to be made from a rectangular piece of cardboard, 7 inches by 3 inches, by cutting equal squares from each corner and turning up the sides.
- Write the volume,  $V$ , as a function of the edge of the square,  $x$ , cut from each corner.
  - Then use the graph of the function to estimate the size of the square that should be cut from each corner and the volume of the largest such box.
21. The sum of two nonnegative numbers is 20. Find the numbers if
- the sum of their squares is as large as possible; as small as possible.
  - one number plus the square root of the other is as large as possible; as small as possible.
22. A rectangle has its base on the x-axis, and its upper two vertices on the parabola  $y = 12 - x^2$ . What is the largest area the rectangle can have, and what are its dimensions?
23. The motions of a particle is given by,  $s = t^3 - 6t^2 + 9t$  ( $t \geq 0$ ), where  $s$  is measured in meters and  $t$  in seconds. A) Find the velocity and the acceleration of the particle at time  $t$ . B) Find when the particle is moving to the left and to the right. C) Find the positions of the particle when the particle is instantaneously at rest. D) Indicate the motion of the particle in a diagram.