Are you ready for Calculus?
See 1.1

1) a $\frac{x^3-9x}{x^2-7x+12} =$

$\frac{x(x+3)(x-3)}{(x-4)(x-3)} =$

$\boxed{\frac{x(x+3)}{x-4}}$

b $\frac{x^2-2x-8}{x^3+x^2-2x} =$

$\frac{(x-4)(x+2)}{x(x^2+x-2)} =$

$\frac{(x-4)(x+2)}{x(x+2)(x-1)} =$

$\boxed{\frac{x-4}{x(x-1)}}$

c $\left(\frac{1}{x}-\frac{1}{5}\right) \cdot \frac{25x^2}{\left(\frac{1}{x^2}-\frac{1}{25}\right)25x^2} =$

$\frac{25x-5x^2}{25-x^2} =$

$\frac{5x(5-x)}{(5+x)(5-x)} =$

$\boxed{\frac{5x}{5+x}}$

2) a $\frac{2}{(\sqrt{3}+\sqrt{2})} \cdot \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \frac{2(\sqrt{3}-\sqrt{2})}{3-2} =$

$= \boxed{2\sqrt{3}-2\sqrt{2}}$

b $\frac{4}{1-\sqrt{5}} \frac{(1+\sqrt{5})}{(1+\sqrt{5})} =$

$= \frac{(4)(1+\sqrt{5})}{1-5} =$

$\frac{\cancel{4}(1+\sqrt{5})}{-\cancel{4}} =$

$\boxed{-1-\sqrt{5}}$

3) $x^6-16x^4 =$

a $x^4(x^2-16) =$

$\boxed{x^4(x+4)(x-4)}$

b $4x^3-8x^2-25x+50 =$

$4x^2(x-2)-25(x-2) =$

$(x-2)(4x^2-25) =$

$\boxed{(x-2)(2x+5)(2x-5)}$

c $8x^3+27 =$

$(2x)^3+3^3 =$

$\boxed{(2x+3)(4x^2-6x+9)}$

4) (a) $f(x) = \frac{5x-3}{2x+1}$

$2x+1 \neq 0$

$x \neq -\frac{1}{2}$

D: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

(b) $f(x) = \sqrt{x-1}$

$x-1 \geq 0$

$x \geq 1$

D: $[1, \infty)$

5) (a) $f(g(1)) = f(0) = 0$

$g(1) = 1^2 - 1 = 0$

$f(0) = \sqrt{0} = 0$

(b) $g(f(0)) = -1$

$f(0) = \sqrt{0} = 0$

$g(0) = 0^2 - 1 = -1$

(c) $f(g(x)) =$

$= \sqrt{(x^2-1)}$

(d) $g(f(x)) =$

$(\sqrt{x})^2 - 1 =$

$= x - 1$

6) $f(x) = 2x+3$

(a)

$$= \frac{2(x+h)+3-(2x+3)}{h}$$

$$= \frac{2x+2h+3-2x-3}{h}$$

$$= \frac{2h}{h} = 2$$

(b)

$$= \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \frac{x+1-(x+h+1)}{(x+1)(x+h+1)} \cdot \frac{1}{h}$$

$$= \frac{-h}{(x+1)(x+h+1)} \cdot \frac{1}{h}$$

$$= \frac{-1}{(x+1)(x+h+1)}$$

(c)

$$= \frac{(x+h)^2 - x^2}{h} =$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{h(2x+h)}{h}$$

$$= 2x+h$$

7.) (a) $\cos 210 = -\frac{\sqrt{3}}{2}$

(b) $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

(c) $\cos \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

(d) $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

(e) $\sin 225° = -\frac{\sqrt{2}}{2}$

8) (a) III

(b) IV

9) (a) $\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

(c) $\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

(d) $\cot = -\sqrt{3}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

1) $\lim_{x\to 2} \frac{x-2}{x^2-x-2} = \boxed{\frac{1}{3}}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	.344828	.334482	.333444	.333222	.332226	.322581

2) $\lim_{x\to 2} \frac{x-2}{x^2-4} = \boxed{\frac{1}{4}}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	.256410	.250627	.250063	.249938	.249377	.243902

9) $\lim_{x\to 3} (4-x) = \boxed{1}$

12) $\lim_{x\to 1} f(x) = \boxed{3}$

$$f(x) = \begin{cases} x^2+2 & x \neq 1 \\ 1 & x = 1 \end{cases}$$

14) $\lim_{x\to 3} \frac{1}{x-3} = \boxed{\text{DNE}}$

$\lim_{x\to 3^-} = -\infty$

$\lim_{x\to 3^+} = +\infty$

Limits do not approach same value ∴ DNE

19) ⓐ $f(1) = \boxed{2}$

ⓑ $\lim_{x \to 1} f(x) = \boxed{DNE}$

ⓒ $f(4) = \boxed{\text{NO solution}}$

ⓓ $\lim_{x \to 4} f(x) = \boxed{2}$

20) a) $f(-2) = \boxed{\text{NO SOL}}$

b) $\boxed{DNE}$ $\lim_{x \to 2^-} \neq \lim_{x \to 2^+}$

c) $\boxed{4}$

d) $\boxed{DNE}$ $\lim_{x \to 0^-} \neq \lim_{x \to 0^+}$

ⓔ $\boxed{\text{NO SOL}}$

f) $\boxed{1/2}$

g) $\boxed{2}$

h) $\boxed{\infty}$

Sect. 1.3

3) $f(x) = x \cos x$

a) $\lim_{x \to 0} f(x) = 0$

b) $\lim_{x \to \frac{\pi}{3}} = \boxed{.5236}$

6) $\lim_{x \to -2} x^3 = \boxed{-8}$

9) $\lim_{x \to -3} (x^2 + 3x) = -3^2 + 3(-3) = 9 - 9 = \boxed{0}$

12) $\lim_{x \to 1} 3x^3 - 2x^2 + 4 = 3 - 2 + 4 = \boxed{5}$

15) $\lim_{x\to 1} \frac{x-3}{x^2+4} = \frac{1-3}{1^2+4} = \boxed{\frac{-2}{5}}$

18) $\lim_{x\to 3} \frac{\sqrt{x+1}}{x^2+4} = \frac{\sqrt{3+1}}{3^2+4} = \frac{\sqrt{4}}{13} = \boxed{\frac{2}{13}}$

21) $\lim_{x\to -4} (x+3)^2 = (-4+3)^2 = -1^2 = \boxed{1}$

24) a) $\lim_{x\to -3} x+7 = -3+7 = \boxed{-4}$

b) $\lim_{x\to 4} x^2 = 4^2 = \boxed{16}$

c) $\lim_{x\to -3} x^2+7 = (-3)^2+7 = 9+7 = \boxed{16}$

27) $\lim_{x\to \frac{\pi}{2}} \sin x = \sin\frac{\pi}{2} = \boxed{1}$

33) $\lim_{x\to \frac{5\pi}{6}} \sin x = \sin\frac{5\pi}{6} = \boxed{\frac{1}{2}}$

30) $\lim_{x\to 1} \sin\frac{\pi x}{2} = \sin\frac{1\pi}{2} = \boxed{1}$

36) $\lim_{x\to 7} \sec\frac{\pi x}{6} = \sec\frac{7\pi}{6} = \frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

39) $\lim_{x\to c} f(x) = 4$

a) $\lim_{x\to c} [(f(x))]^3 = 4^3 = \boxed{64}$

b) $\lim_{x\to c} \sqrt{f(x)} = \sqrt{4} = \boxed{2}$

c) $\lim_{x\to c} [3f(x)] = 3(4) = \boxed{12}$

d) $\lim_{x\to c} [f(x)]^{3/2} = 4^{3/2} = 2^3 = \boxed{8}$

51) $\lim_{x \to -3} \frac{x^2+x-6}{x^2-9} =$

$\lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)(x-3)} =$

$\lim_{x \to -3} \frac{x-2}{x-3} = \frac{-3-2}{-3-3} = \frac{-5}{-6} = \boxed{\frac{5}{6}}$

54) $\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$= \lim_{x \to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} =$

$= \lim_{x \to 0} \frac{x \cdot 1}{x(\sqrt{2+x}+\sqrt{2})} =$

$\frac{1}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$

57) $\lim_{x \to 0} \frac{\frac{1}{(3+x)} - \frac{1}{3}}{x}$

$= \lim_{x \to 0} \frac{\frac{3-(3+x)}{(3+x)(3)}}{x} =$

$= \lim_{x \to 0} \frac{-x \cdot 1}{(3+x)(3)} \left(\frac{1}{x}\right)$

$\lim_{x \to 0} \frac{-1}{(3+x)(3)} = \frac{-1}{3(3)}$

$= \boxed{\frac{-1}{9}}$

60) $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} =$

$= \lim_{\Delta x \to 0} \frac{x^2 - 2x\Delta x - x^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x}{\Delta x} = \boxed{-2x}$

69) $\lim_{x \to 0} \frac{\sin x \,(1-\cos x)}{2x^2} =$ Thm 1.9

$= \lim_{x \to 0} \frac{1}{2}\left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{x}\right)$

$= \left(\frac{1}{2}\right)(1)(0) = \boxed{0}$

72) $\lim_{x \to 0} \frac{\tan^2 x}{x}$

$= \lim_{x \to 0} \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1} \cdot \frac{1}{\cos^2 x}$

$(1)(0)(1) = \boxed{0}$

75) $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\cot x} = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\cos x}{\sin x}}$

$= \lim_{x \to \frac{\pi}{2}} \cos x \left(\frac{\sin x}{\cos x}\right)$

$\lim_{x \to \frac{\pi}{2}} \sin x = \boxed{1}$

3. $\lim_{x \to c} f(x) = 0$

(a) $x \to c^+$

(b) $\lim_{x \to c^-} f(x) = 0$

(c) $\lim_{x \to c} f(x) = 0$

6. $\lim_{x \to c} f(x)$

(a) $x \to c^+ = 0$

(b) $\lim_{x \to c^-} f(x) = 2$

(c) $\lim_{x \to c} f(x) = DNE$

7) $\lim_{x \to 5^+} \frac{(x-5)\,1}{(x+5)(x-5)}$

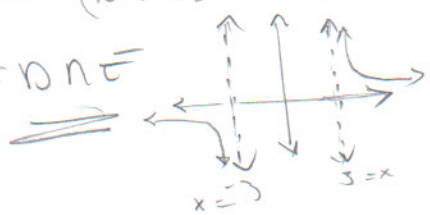
$= \lim_{x \to 5^+} \frac{1}{x+5}$

$= \frac{1}{10}$

9) $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2+9}}$

$\lim_{x \to -3^-} \frac{x\sqrt{x^2-9}}{(x+3)(x-3)} = -\infty$

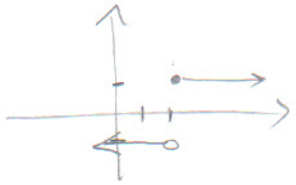
$= DNE$



12. $\lim_{x \to 2^+} \frac{|x-2|}{x-2}$ $\quad |x-2| = \begin{cases} x-2; & x \geq 2 \\ -(x-2); & x < 2 \end{cases}$

(from Right)

$= \frac{x-2}{x-2} = 1$



10. $\lim_{x \to 4^-} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$\lim_{x \to 4^-} \frac{(x-4)\,1}{(x-4)\sqrt{x}+2} = \frac{1}{4}$

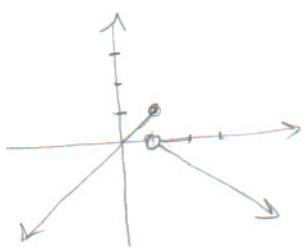
14. $\lim_{\Delta x \to 0^+} \frac{(x+\Delta x)^2 + x + \Delta x - (x^2+x)}{\Delta x}$

$\lim_{\Delta x \to 0^+} \frac{x^2 + 2x\Delta x + \Delta x^2 + x + \Delta x - x^2 - x}{\Delta x}$

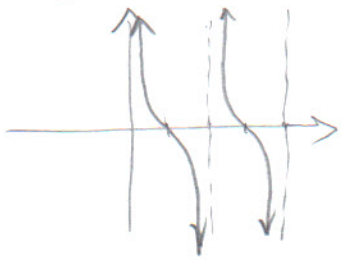
$\lim_{\Delta x \to 0^+} \frac{\Delta x(2x + \Delta x + 1)}{\Delta x}$

$= 2x + 1$

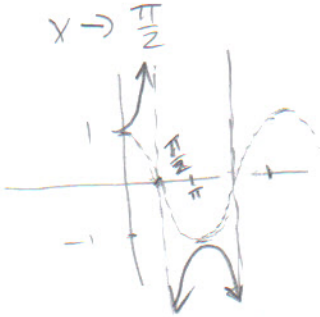
18. $\lim_{x \to 1^+} f(x) = 0$ $\quad f(x) = \begin{cases} x; & x \leq 1 \\ 1-x; & x > 1 \end{cases}$



19. $\lim_{x \to \pi} \cot x$ = DNE



20. $\lim_{x \to \frac{\pi}{2}} \sec x$ = DNE



24. $\lim_{x \to 1} \left(1 - \left[\left[\frac{-x}{2}\right]\right]\right)$

greatest integer $\left| n \leq \frac{-x}{2}\right.$

$= 1 - [[-\frac{1}{2}]]$

$= 1 - (-1)$

$= 2$

22. $\lim_{x \to 2^+} (2x - x) =$

$4 - 2 = 2$

27. $f(x) = \frac{1}{2}[[x]] + x$

nonremovable discontinuities at all x integers.

28. has a removable discontinuity at $x = 1$.

35. $f(x) = 3x - \cos x$

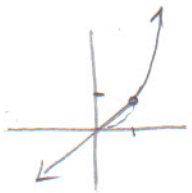
- Continuous

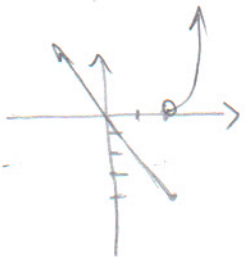
40. $f(x) = \frac{x-3}{x^2-9}$ $(x+3)(x-3)$

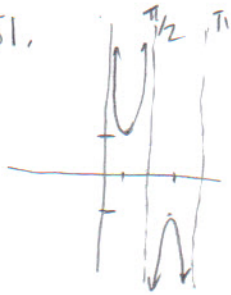
- discontinuity at $x = 3$ & removable
- $x = -3$ nonremovable

41. $f(x) = \frac{x+2}{x^2-3x-10}$ $(x+2)(x-5)$

- discontinuity $x = -2$, removable
- $x = 5$ nonremovable

45.  continuous

48.  nonremovable discontinuity at $x = 2$

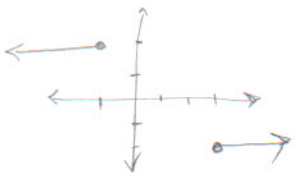
51. 

nonremovable discontinuity at all multiples of $\frac{\pi}{2}$

59. $f(x) = \begin{cases} 2 & x \le -1 \\ ax+b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$

$(-1, 2)$ $(3, -2)$

$y = ax+b$



$2 = -1a + b$

$-(-2 = 3a + b)$

$4 = -4a$

$-1 = a$

$(-1)(-1) + b = 2$

$b = 1$

60. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x-a} & x \neq a \\ 8 & x = a \end{cases}$

$\lim_{x \to a} 8 = 8$

$\lim_{x \to a} \dfrac{x^2 - a^2}{x-a}$

$\lim_{x \to a} \dfrac{(x+a)(x-a)}{(x-a)}$

$= 2a$

$\therefore 2a = 8$

$a = \frac{8}{4}$

$a = 2$

70. $f(x) = x\sqrt{x+3}$

continuous on $[-3, \infty)$

71. $f(x) = \sec \frac{\pi x}{4}$

- continuous on $(-6, -2) \cup (-2, 2) \cup (2, 6) \cup \dots$

61. $f(x) = x^2$ $\quad h(x) = (x-1)^2$

$g(x) = x - 1$ $\quad = x^2 - 2x + 1$

- continuous on $(-\infty, \infty)$

63. $f(x) = \dfrac{1}{x-6}$ $\quad h(x) = \dfrac{1}{x^2+5-6}$

$g(x) = x^2 + 5$ $\quad = \dfrac{1}{x^2-1}$

$x \neq \pm 1$

- nonremovable discontinuity at $x = -1$ & 1

79. $f(x) = x^3 + x - 1$ $\quad [0, 1]$

$x = .68$

$x = .6823$

83 $f(x) = x^2 + x - 1$ $\quad [0, 5]$

$f(c) = 11$

$f(0) = -1$

$f(5) = 25 + 5 - 1 = 29$ $\quad -1 < 11 < 29$

$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, 3$ wet interval

$x = 3$

85. $f(x) = x^3 - x^2 + x - 2 \quad [0,3]$ $\quad f(c) = 4$

$f(0) = -2$

$f(3) = 27 - 9 + 3 - 2$ $\quad -2 < 4 < 19$ ✓ In interval

$= 19$

$x^3 - x^2 + x - 2 = 4$

$x^3 - x^2 + x - 6 = 0$

$x = 2$ $\quad$ & in interval

86. $f(x) = \dfrac{x^2+x}{x-1} \quad \left[\frac{5}{2}, 4\right] \quad f(c) = 6$

$f\left(\frac{5}{2}\right) = \dfrac{\frac{25}{4} + \frac{5}{2}\left(\frac{2}{2}\right)}{\frac{5}{2} - 1\frac{2}{2}} = \dfrac{\frac{35}{4}}{\frac{3}{2}} : \dfrac{35}{\cancel{4}_2} \cdot \dfrac{\cancel{2}}{3} = \dfrac{35}{6} = 5.83\overline{3}$

$f(4) = \dfrac{16+4}{4-1} = \dfrac{20}{3}$

$6.66\overline{6}$

$\frac{35}{6} < 6 < \frac{20}{3}$ ✓ in interval

$\dfrac{x^2+x}{x-1} = 6$

$x^2 + x = 6x - 6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x \neq 2 \quad x = 3$

↑ 2 not in interval $\left[\frac{5}{2}, 4\right]$

1. $\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = \infty$

2. $\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = \infty$

3. $\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = -\infty$

9. $f(x) = \frac{1}{x^2}$

$x^2 \neq 0$

$x = 0$

12. $g(x) = \frac{2+x}{x^2(1-x)}$

$x^2 = 0 \quad 1-x = 0$

$x = 0 \quad x = 1$

15. $g(t) = \frac{t-1}{t^2+1}$

$t^2 \neq -1$

no V.A

16. $h(x) = \frac{2x-3}{x^2-25}$

$x^2 - 25 = 0 \quad x = \pm 5$

$(x+5)(x-5) = 0$

18. $f(x) = \sec \pi x$

$x =$ all odd multiples of $\frac{1}{2}$

21. $f(x) = \frac{x}{x^2+x-2}$

$x^2+x-2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

24. $h(x) = \frac{x^2-4}{x^3+2x^2+x+2}$

$x^2(x+2) + (x+2)$

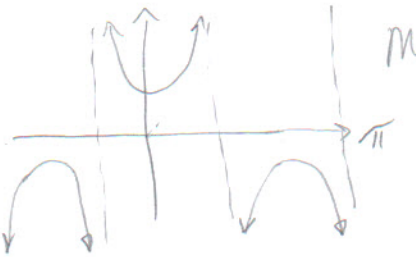
$(x^2+1)(x+2)$

$\therefore h(x) = \frac{(x+2)(x-2)}{(x+2)(x^2+1)}$

$x^2+1 = 0$

$x^2 = -1$ none

27. $s(t) = \frac{t}{\sin t} = t \csc t$



multiples of π except 0

29. $f(x) = \dfrac{x^2+1}{x+1}$

$= \dfrac{(x+1)(x-1)}{(x+1)}$

removov. disc at $x=-1$

30. $f(x) = \dfrac{x^2-6x-7}{x-1}$

$= \dfrac{(x+1)(x-7)}{(x+1)}$

removov. disc at $x=-1$

32. $f(x) = \dfrac{\sin(x+1)}{x+1}$

removov. disc at $x=-1$

33. $\lim_{x\to 1^+} \dfrac{x-3}{x-2} = \dfrac{1-3}{1-2} = \dfrac{-2}{-1}$

$= 2$

36. $\lim_{x\to 4^-} \dfrac{x^2}{x^2+16} = \dfrac{16}{32} = \dfrac{1}{2}$

39. $\lim_{x\to 1} \dfrac{x^2-x}{(x^2+1)(x-1)} =$

$\lim_{x\to 1} \dfrac{x(x-1)}{(x^2+1)(x-1)} = \dfrac{1}{1+1} = \dfrac{1}{2}$

42. $\lim_{x\to 0^-} \left(x^3 - \dfrac{1}{x}\right) = \infty$ (Thm 1.5)

45. $\lim_{x\to\pi} \dfrac{\sqrt{x}}{\csc x} =$

$\lim_{x\to\pi} \dfrac{\sqrt{x}}{\frac{1}{\sin x}} =$

$\lim_{x\to\pi} \sqrt{x}(\sin x)$

$= \sqrt{\pi}\sin\pi = 0$

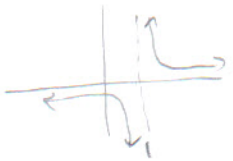
48. $\lim_{x\to\frac{1}{2}} x^2 \tan \pi x$

$= \left(\frac{1}{2}\right)^2 \tan\frac{\pi}{2} = \text{DNE}$

49. $f(x) = \dfrac{x^2+x+1}{x^3-1}$

$\lim_{x\to 1^+} \dfrac{x^2+x+1}{(x-1)(x^2+x+1)}$

$\lim_{x\to 1^+} \dfrac{1}{x-1}$



$= \infty$

50. $f(x) = \dfrac{x^3-1}{x^2+x+1} = \dfrac{(x-1)(x^2+x+1)}{x^2+x+1}$

$\lim_{x\to 1^-} f(x) = 0$

52. $f(x) = \sec\frac{\pi x}{6}$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \sec\frac{\pi x}{6} = \lim_{x\to 3^+} \sec\frac{\pi x}{2}$

$= \sec\frac{\pi}{2} = -\infty$

67. T

68. F $\frac{x^2-1}{x+1} = f(x)$

69. T

10. T