## Multiplication and Division of Rational Expressions Mrs. Snow, Instructor



When given a fraction like $\frac{3}{27}$ we can simplify it by reducing, that is taking out the common factors in the form of $\frac{1}{1}$ or here $\frac{3}{3}$ so: $\frac{3}{27}=\frac{3}{(3)(9)}=\frac{1}{9}$ Why are we doing third grade math??? Well, you have probably figured out by the title of this less where this is going; those rational functions may also be reduced by taking out common factors.

## Factors!!

A number may be created by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number.

Polynomials: we may also be able separate a polynomial into factors of polynomials of lesser degrees that when multiplied together are equivalent to the original polynomial.

When dealing with a RATIONAL POLYNOMIAL, you may only cancel out common factors when the factors are in both the numerator and denominator. ALWAYS FACTOR into simplest form before cancelling out what you think to be a common factor.

$$
\text { Factor the expression: } \begin{aligned}
& \frac{3 x^{2}-3 x-18}{(x+2)} \\
= & \frac{3(x+2)(x-3)}{(x+2)} \\
\therefore= & 3(x-3)
\end{aligned}
$$

Step 1: Factor the numerator and denominator (if needed) into a product of simpler factors.

Are there any common factors?
Step 2: Anything divided by itself is " 1 ", so factor out. Reduce and rewrite.

Can $x=-2$ ?
*RESTRICTIONS!? When there are variables in the denominator, there is always a possibility that setting the variable equal to a specific value will cause the denominator to be equal to zero, hence undefined. So always state the restrictions, that is what values cause the denominator to be equal to zero. Even when the variable is cancelled out with a common factor in the numerator, the restriction still holds true. Look at the original problem, set the denominator equal to zero, and solve!

Factor and state restrictions *
$\frac{x^{2}+x-12}{x^{2}+5 x-24}$

$$
\frac{2 x^{2}-8 x-64}{x^{3}+10 x^{2}+24 x}
$$

Warning: The common temptation at this point is to try to cancel out common terms. NEVER EVER NEVER cancel out individual terms that are either added or subtracted. Whenever you have terms added together, there are invisible parentheses around them, like this: $\frac{(x+4)}{(x+8)}$. You can only cancel out factors (that is, entire expressions contained within parentheses), not terms that are separated by arithmetic signs.

Think: $\frac{4+9}{1+9}=\frac{13}{10}=1.3$ NOT!!!! $\quad \frac{4+\beta}{1+\beta}=\frac{4}{1}=4$ IT IS ILLEGAL TO CANCEL OUT INDIVIDUAL TERMS IN A FRACTION UNLESS IT IS A FACTOR!! YOU WOULD NOT DO THIS in a simple fraction, THEREFORE DO NOT DO THIS WITH RATIONAL EXPRESSIONS!

SO NEVER DO THIS! $\frac{\mid+3}{\mid+6}=\frac{3}{6}=\frac{1}{2}$ absolutely no partial credit is given for this type of error!
Remember


## Multiplication of Rational Expressions

What is possible with a simple numerical fraction is also possible with rational expressions. As with fractions, multiplication will be easier if you first simplify, then multiply:
$\frac{12}{8} \cdot \frac{64}{3<}=\frac{1}{1} \cdot \frac{8}{3}=\frac{8}{3}$ factor out an 8 and a 12. Here, by the time you reduce, you have nothing to multiply!
Factor and simplify the following expressions, restrictions??

$$
\frac{2 x^{2}+7 x+3}{x-4} \cdot \frac{x^{2}-16}{x^{2}+8 x+15}
$$

$$
\frac{a^{2}-4}{a^{2}-1} \cdot \frac{a+1}{a^{2}+2 a}
$$

## Division of Rational Expressions

Remember how you learned to divide fractions in grade school? Keep...Change...Flip


We can follow the same process for division with rational expressions. NOTE: Any restrictions on the variables also applies to the divisor's numerator! Why? Look below

$$
\begin{aligned}
& \text { Can } a=-1 ? \\
& \qquad \frac{a^{2}+2 a-15}{a^{2}-16} \div \frac{a+1}{3 a-12}
\end{aligned}
$$

$$
\frac{(4-x)}{(3 x+2)(x-2)} \div \frac{5(x-4)}{(x-2)(7 x-5)}
$$

