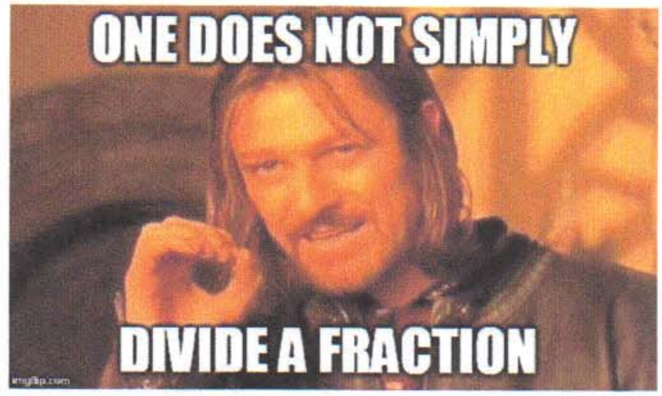
Multiplication and Division of Rational Expressions
Mrs. Snow, Instructor



When given a fraction like $\frac{3}{27}$ we can simplify it by reducing, that is taking out the common factors in the form of $\frac{1}{1}$ *or here* $\frac{3}{3}$ so: $\frac{3}{27} = \frac{3}{(3)(9)} = \frac{1}{9}$ Why are we doing third grade math??? Well, you have probably figured out by the title of this less where this is going; those rational functions may also be reduced by taking out common factors.

Factors!!
A number may be created by multiplying two or more other numbers together. The numbers that are multiplied together are called **factors** of the final number.

Polynomials: *we may also be able separate a polynomial into factors of polynomials of lesser degrees that when multiplied together are equivalent to the original polynomial.*

When dealing with a RATIONAL POLYNOMIAL, you may only cancel out common factors when the factors are in both the numerator and denominator. ***ALWAYS FACTOR into simplest form before cancelling out what you think to be a common factor.***

Factor the expression: $\frac{3x^2-3x-18}{(x+2)}$

$= \frac{3(x+2)(x-3)}{(x+2)}$

$\therefore = 3(x-3)$

Step 1: Factor the numerator and denominator (if needed) into a product of simpler factors.

Are there any common factors?

Step 2: Anything divided by itself is "1", so factor out. Reduce and rewrite.

***RESTRICTIONS!?** When there are variables in the denominator, there is always a possibility that setting the variable equal to a specific value will cause the denominator to be equal to zero, hence undefined. So always state the restrictions, that is what values cause the denominator to be equal to zero. *Even when the variable is cancelled out with a common factor in the numerator, the restriction still holds true. Look at the original problem, set the denominator equal to zero, and solve!*

Factor and state restrictions *

$\frac{x^2+x-12}{x^2+5x-24} = \frac{(x+4)\cancel{(x-3)}}{(x+8)\cancel{(x-3)}}$

$= \frac{x+4}{x+8}, x \neq -8, 3$

$x+8 \neq 0$
$x \neq -8$
$x-3 \neq 0$
$x \neq 3$

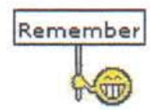
$\frac{-27x^3y}{9x^4y}$

$\frac{x^2+10x+25}{x^2+9x+20} = \frac{\cancel{(x+5)}(x+5)}{\cancel{(x+5)}(x+4)}$ $= \frac{x+5}{x+4}, \quad x \neq -5, -4$	$\frac{2x^2-8x-64}{x^3+10x^2+24x} = \frac{2(x^2-4x-32)}{x(x^2+10x+24)}$ $= \frac{2(x-8)\cancel{(x+4)}}{x\cancel{(x+4)}(x+6)} = \frac{2(x-8)}{x(x+6)};$ $x \neq 0, -4, -6$

Make sure to state restrictions on the examples above.

Warning: The common temptation at this point is to try to cancel out common terms. *NEVER EVER NEVER* cancel out individual terms that are either added or subtracted. Whenever you have terms added together, there are invisible parentheses around them, like this: $\frac{(x+4)}{(x+8)}$. You can only cancel out factors (that is, entire expressions contained within parentheses), not terms that are separated by arithmetic signs.

Think: $\frac{4+9}{1+9} = \frac{13}{10} = 1.3$ **NOT!!!!** $\frac{4+\cancel{9}}{1+\cancel{9}} = \frac{4}{1} = 4$ ***IT IS ILLEGAL TO CANCEL OUT INDIVIDUAL TERMS IN A FRACTION UNLESS IT IS A FACTOR!! YOU WOULD NOT DO THIS in a simple fraction, THEREFORE DO NOT DO THIS WITH RATIONAL EXPRESSIONS!***



SO NEVER DO THIS! $\frac{\cancel{x}+3}{\cancel{x}+6} = \frac{3}{6} = \frac{1}{2}$ ***absolutely no partial credit is given for this type of error!***

Multiplication of Rational Expressions

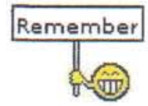
What is possible with a simple numerical fraction is also possible with rational expressions. As with fractions, multiplication will be easier if you first simplify, then multiply:

$\frac{\cancel{12}}{\cancel{8}} \cdot \frac{\cancel{64}}{\cancel{36}} = \frac{1}{1} \cdot \frac{8}{3} = \frac{8}{3}$ factor out an 8 and a 12. Here, by the time you reduce, you have nothing to multiply!

Factor and simplify the following expressions, restrictions??

$\frac{2x^2+7x+3}{x-4} \cdot \frac{x^2-16}{x^2+8x+15} =$ $\frac{(2x+1)\cancel{(x+3)}}{\cancel{(x-4)}} \cdot \frac{(x+4)\cancel{(x-4)}}{\cancel{(x+3)}(x+5)} =$ $\frac{(2x+1)(x+4)}{(x+5)} \quad x \neq 4, -3, -5$	$\frac{a^2-4}{a^2-1} \cdot \frac{a+1}{a^2+2a} =$ $\frac{\cancel{(a+2)}(a-2)}{\cancel{(a+1)}(a-1)} \cdot \frac{\cancel{(a+1)}}{a\cancel{(a+2)}} =$ $\frac{(a-2)}{a(a-1)}, \quad x \neq \pm 1, 0, -2$

Division of Rational Expressions



Remember how you learned to divide fractions in grade school? ***Keep…Change…Flip***
We can follow the same process for division with rational expressions. *NOTE: Any restrictions on the variables also applies to the divisor's numerator! Why? Look below*

Can $a = -1$? NO!!

$$\frac{a^2+2a-15}{a^2-16} \div \frac{a+1}{3a-12} =$$

$$\frac{(a+5)(a-3)}{(a+4)(a-4)} \div \frac{(a+1)}{3(a-4)} =$$

$$\frac{(a+5)(a-3)}{(a+4)\cancel{(a-4)}} \cdot \frac{3\cancel{(a-4)}}{(a+1)} =$$

$$\frac{3(a+5)(a-3)}{(a+4)(a+1)} \qquad a \neq \pm 4, -1$$

$$\frac{(4-x)}{(3x+2)(x-2)} \div \frac{5(x-4)}{(x-2)(7x-5)}$$

$$\frac{(4-x)}{(3x+2)(x-2)} \cdot \frac{(x-2)(7x-5)}{5(x-4)}$$

Almost but not quite! factor out a "-1"

$$\frac{-1\cancel{(x-4)}}{(3x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(7x-5)}{5\cancel{(x-4)}}$$

$$= \frac{-(7x-5)}{5(3x+2)} \qquad x \neq -\frac{2}{3}, 2, 4, \frac{5}{7}$$