Addition and Subtraction of Rational Expressions
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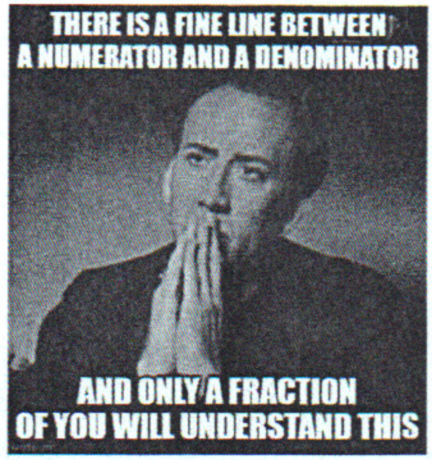

Last section we saw how we could apply techniques for multiplying and dividing fractions to rational expressions. Well, we also can apply addition and subtractions techniques to rational expressions. So what are the steps in adding two fractions?

$\frac{2}{3}+\frac{3}{4}+\frac{1}{6}$

3: 3
4: $2 \cdot 2$
6: $2 \cdot 3$

LCM:
need *one:* 3
need two: $\cdot$ 2s
have these

LCM: $2 \cdot 2 \cdot 3 = 12$

$$\frac{4}{4}\cdot\frac{2}{3}+\frac{3}{3}\cdot\frac{3}{4}+\frac{2}{2}\cdot\frac{1}{6}=\frac{8}{12}+\frac{9}{12}+\frac{2}{12}=\frac{19}{12}$$

1. factor each denominator term
2. find the least common multiple of the denominators (what is the smallest number that the denominators are all factors of?)
3. multiply each term by "1" so that they have common denominators
4. add and simplify

Note: In this example the denominator "12" is known as the least common denominator or LCD

For rational expressions it is a bit more complicated in that finding the LCM involves factorization of a polynomial, but you have the tools to do this!

Find the least common multiple:

x^2-9 *and* x^2+6x+9	$3x^2-9x-30$ *and* $6x-30$
$=(x+3)(x-3)$ $\quad (x+3)(x+3)$	$3(x^2-3x-10)$ $\quad 6(x-5)$
need 1 $\quad$ need 2	$3(x-5)(x+2)$ $\quad 3(2)(x-5)$
$LCM=(x-3)(x+3)(x+3)$	LCM: $(3)(2)(x-5)(x+2)$

Add/subtract:

$$\frac{2}{x-3}-\frac{4x}{x+4}$$ Negative Distribute

$$=\frac{(x+4)}{(x+4)}\cdot\frac{(2)}{(x-3)}-\frac{4x}{(x+4)}\cdot\frac{(x-3)}{(x-3)}$$

$$=\frac{\cancel{2x}+8-4x^2+12x}{(x+4)(x-3)}$$

$$=\frac{-4x^2+14x+8}{(x+4)(x-3)}$$

$$\left(\frac{2y}{2y}\right)\frac{2x+1}{x^2y}+\frac{3}{2y^2}\left(\frac{x^2}{x^2}\right)=$$

$$\frac{4xy+2y+3x^2}{2x^2y^2}$$

$$\frac{1}{x^2-4x-12}+\frac{3x}{4x+8}$$

$$=\left(\frac{4}{4}\right)\frac{1}{(x-6)(x+2)}+\frac{3x}{4(x+2)}\left(\frac{x-6}{x-6}\right)$$

$$=\frac{4+3x^2-18x}{4(x-6)(x+2)}$$

$$=\frac{3x^2-18x+4}{4(x-6)(x+2)}$$

$$\left(\frac{x+2}{x+2}\right)\frac{x}{3x^2-9x+6}-\frac{2x+1}{3x^2+3x-6}\left(\frac{x-2}{x-2}\right)$$

$3(x^2-3x+2)$ $\quad 3(x^2+x-2)$

$3(x-2)(x-1)$ $\quad 3(x+2)(x-1)$

$$=\frac{x^2+2x-2x^2+4x-x+2}{3(x+2)(x-2)(x-1)}$$

$$=\frac{-x^2+5x+2}{3(x+2)(x-2)(x-1)}$$

Complex Fractions

A fraction can be further complicated by both the numerator and denominator being fractions. When this is the case, we have a **complex fraction.** There are two methods for simplifying complex fractions. The first method is what we have just been studying

1. find the LCD for all the denominators.
2. multiply main numerator and denominator by the LCD
3. and simplify

The second method is sort of a hybrid of LCD and division. We first look at the numerator and denominator separately, combine the terms in the numerator and combine the terms in the denominator so that you get a "fraction over a fraction":

1. For the numerator, find the LCD of the terms and combine the numerator terms as we have done with LCD problems.
2. For the denominator, find the LCD of the terms and combine the denominator terms just as we have done with all other LCD problems.
3. Now with simplified numerator and denominator, flip the denominator (reciprocal) and multiply it by the numerator.
4. Simplify and note any restrictions on the variables.

So let's look at a couple problems and do some problems:

$$\frac{\frac{1}{x}+\frac{2}{3x}}{\frac{3}{2y}+\frac{5}{y}} \quad \frac{(3)(2)(x)(y)}{(3)(2)(x)(y)}$$

method 1:
multiple denominators
LCD: $= (3)(2)(x)(y)$

$$\frac{(3)(2)(x)(y)\frac{1}{\cancel{x}} + \frac{2}{\cancel{3x}}(\cancel{3})(2)(x)(y)}{(3)(\cancel{2})(x)(\cancel{y})\frac{3}{\cancel{2y}} + \frac{5}{\cancel{y}}(3)(2)(x)(\cancel{y})} =$$

$=$

$$\frac{6y+4y}{9x+30x} = \frac{10y}{39x}$$

$$\frac{\frac{x}{x+2}}{\frac{3x^2}{x-4}} \quad \div$$

method 2:
works best when it is in a form for a "fraction over a fraction" or keep-change-flip:

$$\frac{x}{(x+2)} \div \frac{3x^2}{(x-4)} =$$

$$\frac{\cancel{x}}{(x+2)} \times \frac{(x-4)}{3x^{\cancel{2}}} =$$

$$\frac{x-4}{3x(x+2)} \overset{or}{=} \frac{(x-4)}{3x^2+6x}$$

Look at all the denominator to find LCD:

$$\frac{\frac{1}{x}+\frac{3}{y}}{\frac{5}{y}+\frac{4}{x}} \quad \frac{(xy)}{(xy)} =$$

LCD = xy

$$\frac{\cancel{x}y\left(\frac{1}{\cancel{x}}\right)+\left(\frac{3}{\cancel{y}}\right)(x\cancel{y})}{x\cancel{y}\left(\frac{5}{\cancel{y}}\right)+\left(\frac{4}{\cancel{x}}\right)(xy)} =$$

$$\frac{y+3x}{5x+4y} =$$

$$\frac{\frac{2xy^2}{5}}{\frac{6x^3y}{5}} \quad \div$$

Keep - Change - flip

$$\frac{\cancel{2}\cancel{x}y^{\cancel{2}\,1}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{3}{\cancel{6}}x^{\cancel{3}\,2}\cancel{y}} =$$

$$= \frac{y}{3x^2}$$

Example of a more advanced problem:

method 1:

LCD = (x)(x+1)(x-1)

$$\frac{\frac{x-2}{x} - \frac{2}{x+1}}{\frac{3}{x-1} - \frac{1}{x+1}} \quad \times 1 \quad =$$

$$\frac{\left(\frac{x-2}{x} \cdot - \frac{2}{x+1}\right)\cdot\left(x(x+1)(x-1)\right)}{\left(\frac{3}{x-1} - \frac{1}{x+1}\right)\cdot\left(x(x+1)(x-1)\right)} =$$

$$\frac{(x+1)(x-1)(x-2) - x(x-1)(2)}{3x(x+1) - x(x-1)} =$$

$$\frac{x^3 - 2x^2 - x + 2 - 2x^2 + 2x}{3x^2 + 3x - x^2 + x} =$$

$$= \frac{x^3 - 4x^2 + x + 2}{2x^2 + 4x}$$

1. LCD for all the denominators: $x(x+1)(x-1)$

2.Look at the big fraction: multiply the big fraction by "1" our common denominator; remember distribution!!

3. When we multiply each term, cancel out the common factors; you should get rid of the denominators in your numerator and the denominator in the denominator! Whew!

4. Simplify the numerator and denominator

5. Combine like terms and we're done

method 2: combine terms in numerator and denominator then flip and multiply

$$\frac{\frac{(x+1)}{(x+1)}\cdot\frac{x-2}{x} - \frac{2}{x+1}\cdot\frac{(x)}{(x)}}{\frac{(x+1)}{(x+1)}\cdot\frac{3}{x-1} - \frac{1}{x+1}\cdot\frac{(x-1)}{(x-1)}} =$$

numerator LCM = (x)(x+1)

denominator LCM = (x+1)(x-1)

$$\frac{\frac{x-2}{x}\,\frac{(x+1)}{(x+1)} - \frac{2}{x+1}\,\frac{x}{x}}{\frac{3}{x-1}\,\frac{(x+1)}{(x+1)} - \frac{1}{x+1}\,\frac{(x-1)}{(x-1)}} = \frac{\frac{x^2-3x-2}{x(x+1)}}{\frac{2x+4}{(x-1)(x+1)}}$$

$$\frac{\frac{x^2-3x-2}{x(x+1)}}{\frac{2x+4}{(x-1)(x+1)}}\,(\div) =$$

$$\frac{x^2-3x-2}{x\cancel{(x+1)}}\cdot\frac{(x-1)\cancel{(x+1)}}{2x+4} =$$

$$= \frac{x^3 - 4x^2 + x + 2}{2x^2 + 4x}$$

Here we focus on the numerator separately from the denominator.

1. Find the LCD for the terms in numerator

2. Multiply and simply

3. Do the same for the denominator

4. combine and then do the: keep-change-flip

One way may seem easier to some students and the other say easier for others. You choose, but remember, you also need to follow directions!