Precalculus
Lesson 12.1: Sequences
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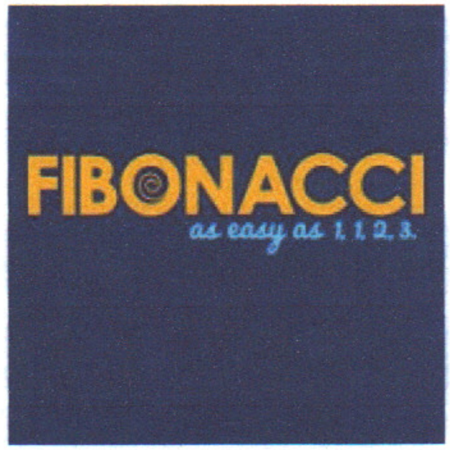

When we hear the word **sequence** we most likely think of a "sequence of events;" something that happens first, then second, and so on. Hey in math it is the same idea. Here a sequence deals with numerical outcomes that are first, second, and so on.

A **sequence** is a function f whose domain is the set of positive integers. The values $f(1), f(2), f(3)$, ... are called terms. Dictionary states: a particular in which related things follow each other.

Write down the first six terms of the following sequence.

$$\{a_n\} = \left\{\frac{n-1}{n}\right\}$$

$\{a_1\} = \left\{\frac{1-1}{1}\right\} = 0$

$a_2 = \frac{2-1}{2} = \frac{1}{2}$

$a_3 = \frac{3-1}{3} = \frac{2}{3}$

$a_4 = \frac{3}{4}$

$a_5 = \frac{4}{5}$

$a_6 = \frac{5}{6}$

Do you see a pattern?

remember to reduce!

Write down the first six terms of the following sequence.

$$\{b_n\} = \left\{(-1)^{n+1}\left(\frac{2}{n}\right)\right\}$$

$b_1 = (-1)^{1+1}\left(\frac{2}{1}\right) = \frac{2}{1}$

$b_2 = (-1)^{2+1}\left(\frac{2}{2}\right) = -\frac{2}{2} = -1$

$b_3 = -1^{3+1}\left(\frac{2}{3}\right) = \frac{2}{3}$

$b_4 = -\frac{2}{4} = \frac{-1}{2}$

$b_5 = \frac{2}{5}$

$b_6 = \frac{-2}{6} = -\frac{1}{3}$

follow the pattern

Write down the first six terms of the following sequence.

$$\{c_n\} = \begin{pmatrix} n \ \ if\ n\ is\ even \\ \frac{1}{n}\ if\ n\ is\ odd \end{pmatrix} = \frac{1}{1}, \ 2, \ \frac{1}{3}, \ 4, \ \frac{1}{5}, \ 6$$

$c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6$

Determining a Sequence from a Pattern

Number the terms and see what happens between each term:

$n = 1 \quad 2 \quad 3 \quad 4$

(a) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \ldots$

$a_n = \frac{e^n}{n}$

$n = 1 \quad 2 \quad 3 \quad 4$

(b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

$\frac{1}{1}$

$3^0 \quad 3^1 \quad 3^2 \quad 3^3$

$b_n = \frac{1}{3^{n-1}}$

$n = 1 \quad 2 \quad 3 \quad 4$

(c) $1, 3, 5, 7, \ldots$ — $2(4)-1$

$2(2)-1 \quad 2(3)-1$

$c_n = 2n-1$

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5$

(d) $1, 4, 9, 16, 25, \ldots$

$d_n = n^2$

$1 \quad 2 \quad 3 \quad 4 \quad 5$

(e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

$e^n = \left(\frac{1}{n}\right)\left(-1^{n+1}\right)$

Some sequences involve a special product called a *factorial:*

A factorial is a product of every integer from 1 to the number n.

$$n! = n(n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \qquad for\ n \geq 2$$

where:

$$0! = 1 \qquad and\ 1! = 1$$

Solve:

9! $= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{362,880}$

On the calculator:
Type number,
MATH ▶ ▶ ▶ PRB
4: ! ENTER

$\frac{12!}{10!} = \frac{12 \cdot 11 \cdot \cancel{10} \cdot \cancel{9} \cdot \cancel{8!}}{\cancel{10} \cdot \cancel{9} \cdot \cancel{8!}} = \boxed{132}$

Wait! we don't have to expand all the way to 1 !!

$\frac{3!\,7!}{4!} = \frac{3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = \boxed{1260}$

A Sequence Defined by a Recursive Formula

A second way of defining a sequence is to assign a value to the first term and specify the nth term by a formula or equation that involves on or more of the terms preceding it. The sequence is defined **recursively**, and the formula is a **recursive formula**.

Write the first 5 terms of the recursive sequence

$$u_1 = 1,\ u_2 = 1,\ u_n = u_{n-2} + u_{n-1}$$

$u_1 = 1$

$u_2 = 1$

$u_3 = u_{3-2} + u_{3-1} = u_1 + u_2 = 1 + 1 = 2$

$u_4 = u_{4-2} + u_{4-1} = u_2 + u_3 = 1 + 2 = 3$

$u_5 = u_{5-2} + u_{5-1} = u_3 + u_4 = 2 + 3 \quad 5$

$u_6 = u_4 + u_5 = \quad = 8$

Pattern? Add previous 2 terms

Known as the Fibonacci Sequence

Sigma Notation: a short cut notation to indicate the sum of some or all of the terms of a sequence:

Given a sequence

$$a_1, a_2, a_3, a_4, \ldots a_n.$$

we can write the sum of the first n terms using **summation notation**, or **sigma notation.** The notation derives its name from the Greek Letter **Σ.** This corresponds to our S for "sum." The following notation is used

The sum of a specified set of terms of a sequence

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots a_n$$

k is called the index of summation, it is **the starting number for the sequence**.

Write out each sum

*$\sum_{k=1}^{10} \frac{1}{k} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+$

*$\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$

*Adding up the first, K=1 through tenth → 10 terms

$\sum_{k=1}^{n} k! = 1!+2!+3!+\dots n!$

Show the patter with the first 3 or 4 terms and the final term

Express each sum using summation notation

$1^2 + 2^2 + 3^2 + \cdots + 9^2$

$\sum_{K=1}^{9} = K^2$

(Adding 1st to 9th terms)

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}}$

$\sum_{K=1}^{n} \frac{1}{2^{K-1}}$

final term shows Summation formula

The following sums are natural consequences of properties of the real numbers. These are useful for adding terms of a sequence algebraically:

Properties of Sequences

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then:

$$\sum_{k=1}^{n}(ca_k) = ca_1 + ca_2 + \cdots + ca_n = c(a_1 + a_2 + \cdots + a_n) = c\sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k, \quad \text{where } 0 < j < n$$

Find the sums:

$\sum_{k=1}^{5} k^2 = 1^2+2^2+3^2+4^2+5^2$

$= 1+4+9+16+25$

$10+20+25 \quad = \underline{55}$

$$\sum_{j=3}^{5}\frac{1}{j} = \frac{20 \cdot 1}{20 \cdot 3} + \frac{15 \cdot 1}{15 \cdot 4} + \frac{1}{5}\cdot\frac{12}{12} =$$

common denominator

$$\frac{20+15+12}{60} = \frac{47}{60}$$

$$\sum_{i=5}^{10} i = 5 + 6 + 7 + 8 + 9 + 10$$

$$15 + 15 + 15 = \underline{\underline{45}}$$

$$\sum_{i=1}^{6} 2 = \underset{1}{2} + \underset{2}{2} + \underset{3}{2} + \underset{4}{2} + \underset{5}{2} + \underset{6}{2} = 6(2) = \underline{\underline{12}}$$

$i \Rightarrow$ 1 2 3 4 5 6

each term in the sequence is in this case 2