Calculus
Lesson 3.5: Limits at Infinity
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When you find there are limits at infinity

In this section we will look at the "end behavior" of a function on an infinite interval.

DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L.$$

THEOREM 3.10 LIMITS AT INFINITY

If r is a positive rational number and c is any real number, then

$$\lim_{x \to \infty} \frac{c}{x^r} = 0.$$

Furthermore, if x^r is defined when $x < 0$, then

$$\lim_{x \to -\infty} \frac{c}{x^r} = 0.$$

Finding a Limit at infinity

$$\lim_{x \to \infty} \left(5 - \frac{2}{x^2}\right).$$

$$\lim_{x \to \infty} 5 - \lim_{x \to \infty} \frac{2}{x^2}$$

0

$= 5$

$$\lim_{x \to \infty} \frac{2x - 1}{x + 1}. \quad = 2$$

degree of num = denom

GUIDELINES FOR FINDING LIMITS AT $\pm\infty$ OF RATIONAL FUNCTIONS

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

In regard to #3, when you encounter an indeterminate form you should divide the numerator and denominator by the **highest power of x in the denominator**.

$$\lim_{x\to\infty}\frac{2x+5}{3x^2+1} = 0 \qquad \lim_{x\to\infty}\frac{2x^2+5}{3x^2+1} = \frac{2}{3} \qquad \lim_{x\to\infty}\frac{2x^3+5}{3x^2+1}$$

num deg < denom deg

degrees =

$$\lim_{x\to\infty}\frac{\frac{2x^3}{x^2}+\frac{5}{x^2}\to 0}{\frac{3x^2}{x^2}+\frac{1}{x^2}\to 0}$$

$$= \lim_{x\to\infty}\frac{2x}{3} = \frac{\infty}{3}$$

$$= \infty$$

A Function with Two Horizontal Asymptotes

a. $\lim_{x\to\infty}\frac{3x-2}{\sqrt{2x^2+1}}$

$x>0 \quad x=\sqrt{x^2}$

$$\lim_{x\to\infty}\frac{\frac{3\cancel{x}}{\cancel{x}}-\frac{2}{x}\to 0}{\sqrt{\frac{2\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}\to 0}}$$

$$= \frac{3}{\sqrt{2}}$$

b. $\lim_{x\to-\infty}\frac{3x-2}{\sqrt{2x^2+1}}$

$x<0 \quad x=-\sqrt{x^2}$

$$\lim_{x\to\infty}\frac{\frac{3\cancel{x}}{\cancel{x}}-\frac{2}{x}\to 0}{-\sqrt{\frac{2\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}\to 0}}$$

$$= \frac{-3}{\sqrt{2}}$$

Limits Involving Trig Functions

$\lim_{x\to\infty}\sin x$ = DNE

Sine oscillates between -1 and 1

so: DNE

$\lim_{x\to\infty}\frac{\sin x}{x} = 0$

Since Sine oscillates:

$-1 \le \sin x \le 1$

divide by x —

$0 \leftarrow \frac{-1}{\cancel{x}} \le \frac{\sin x}{x} \le \frac{1}{\cancel{x}} \rightarrow 0$

$\lim_{x\to 0}\frac{-1}{x}$ $\qquad$ $\lim_{x\to 0}\frac{1}{x}$

Finding Infinite Limits at Infinity

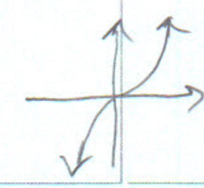
Many functions do not approach a finite limit as x increases or decreases without bound. As an example, polynomial functions do not have a finite limit at infinity.

$\lim_{x\to\infty} x^3 = \infty$

x increases without bound

$\lim_{x\to-\infty} x^3 = -\infty$

x decreases without bound



When evaluating a rational function, use long division to rewrite the improper rational function as the sum of a polynomial and a rational function. From precal: Divide the improper fraction: The quotient is the equation of an oblique asymptote.

$\lim_{x\to\infty}\frac{2x^2-4x}{x+1}$

$= \lim_{x\to\infty} 2x-6 + \frac{6}{\cancel{x}+1} \rightarrow 0$

$= \infty$

$$\lim_{x\to\infty}\frac{\frac{2x^2}{x}-\frac{4}{x}}{\frac{x}{x}+\frac{1}{x}} = \infty$$

Also you can divide by the degree in the denominator

$\lim_{x\to-\infty}\frac{2x^2-4x}{x+1}$

$\lim_{x\to-\infty} 2x+6 + \frac{6}{\cancel{x}+1} \rightarrow 0$

$$\begin{array}{r|l} & 2x-6 + \frac{6}{x+1} \\ x+1 & 2x^2-4x \\ & -2x^2 \mp 2x \\ \hline & -6x \\ & \mp -6x \pm 6 \\ \hline & 6 \text{ (remainder)} \end{array}$$