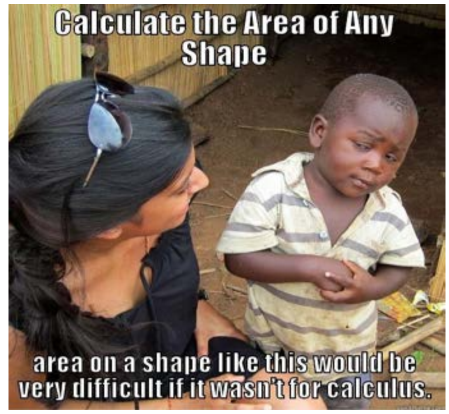
Calculus
Lesson 4.2: Area
Mrs. Snow, Instructor



In this section we will: review sigma notation and evaluate a sum, understand the concept of area, approximate the area of a plane region and find the area of a plane region using limits.

SIGMA NOTATION

The sum of n terms $a_1, a_2, a_3, \ldots, a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where i is the **index of summation,** a_i is the ***i*th term** of the sum, and the **upper and lower bounds of summation** are n and 1.

Examples of Sigma Notation

a. $\sum_{i=1}^{6} i =$

b. $\sum_{i=0}^{5} (i + 1) =$

c. $\sum_{j=3}^{7} j^2 =$

d. $\sum_{k=1}^{n} \frac{1}{n}(k^2 + 1) =$

e. $\sum_{i=1}^{n} f(x_i)\,\Delta x =$

From precalculus we know:

$$\textbf{1. } \sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

$$\textbf{2. } \sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

THEOREM 4.2 SUMMATION FORMULAS

$$\sum_{k=1}^{n} c = nc$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

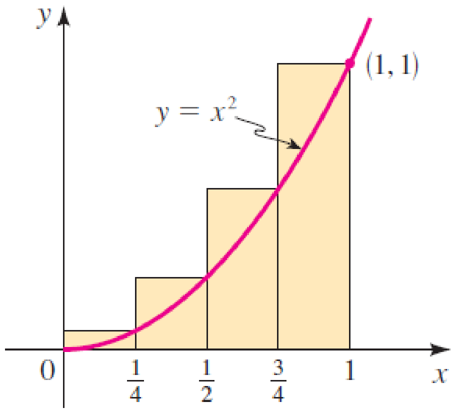
Evaluating a Sum

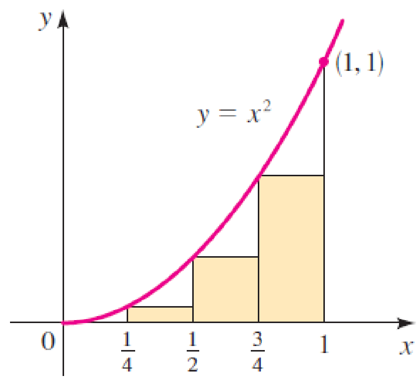
Evaluate

$$\sum_{k=1}^{n} \frac{k+1}{n^2}$$

In geometry we found area of polygons. We had set formulas such as the area of a rectangle is length times width. A triangular area is found by calculating ½ the length of the base times the height, and so on. Calculus is used to deal with area problems that have regions containing curved boundaries. Here we can go back to our simple formula for the area of a rectangle and use it to estimate the area of a region under a curve.

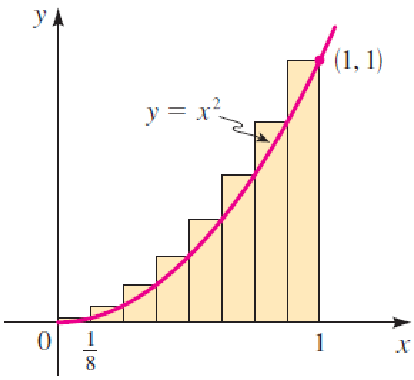
Estimating an Area Using Rectangles: Use rectangles to estimate the area under the parabola $y = x^2$, from 0 to 1.





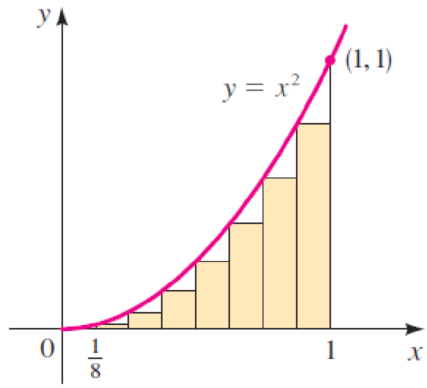
Same problem….smaller rectangles

Overestimate of area.



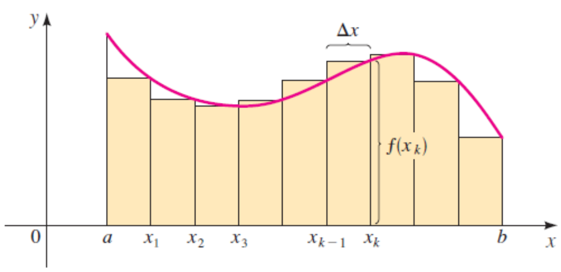
(b) Using right endpoints

Underestimate of area.



(a) Using left endpoints

The area A of the region S that lies under the graph of a continuous function f is the limit of the sum of the areas of the approximating rectangles: ***use right endpoints***.



AREA=(height) x (width)

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(x_k\right)\Delta x$$

Δx **is the width** of an approximating rectangle,
x_k **is the right endpoint** of the kth **rectangle**
$f(x_k)$ **is its height**.

$\boldsymbol{n}$ *rectangles*
region from $x = a$ *to* $x = b$

$\boldsymbol{width}$: $\boldsymbol{\Delta x = \frac{b-a}{n}}$

$\boldsymbol{right\ endpoint}$: $\boldsymbol{x_k = a + k\Delta x}$

$\boldsymbol{height}$: $\boldsymbol{f(x_k) = f(a + k\Delta x)}$

Finding the Area by the Limit Definition

Find the area of the region bounded by the function and the vertical lines x=0 and x=1.

$f(x) = x^3$

Find the area of the region bounded by the function and the vertical lines x=1 and x=2.

$f(x) = 4 - x^2$