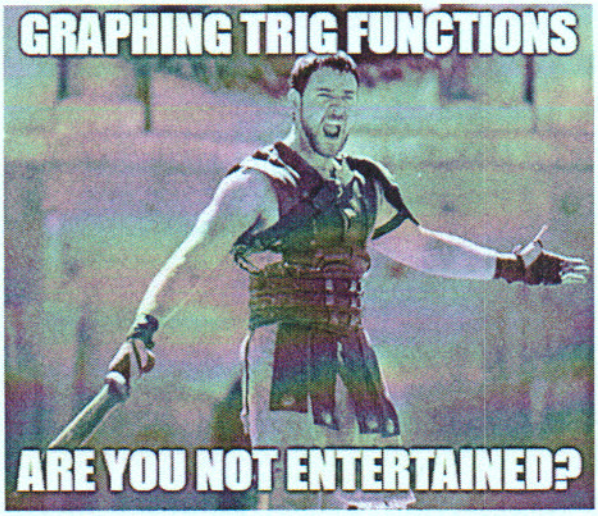
Precalculus

Lesson 6.5 Part 1: Graphs of the Cosecant and Secant Functions

Mrs. Snow, Instructor



The same process used for sine and cosine may be followed for these trig functions.

Cosecant and Secant Functions

The cosecant and secant functions are **reciprocal functions.** These functions are graphed by first graphing sine or cosine. Where the sine and cosine functions have zeros the reciprocals of cosecant secant functions will be undefined, hence, asymptotes.

To graph $\csc\theta$ First graph $\sin\theta$ ①

Where $\sin\theta = 0$ $\csc\theta$ is undefined ∴ Asymptotes

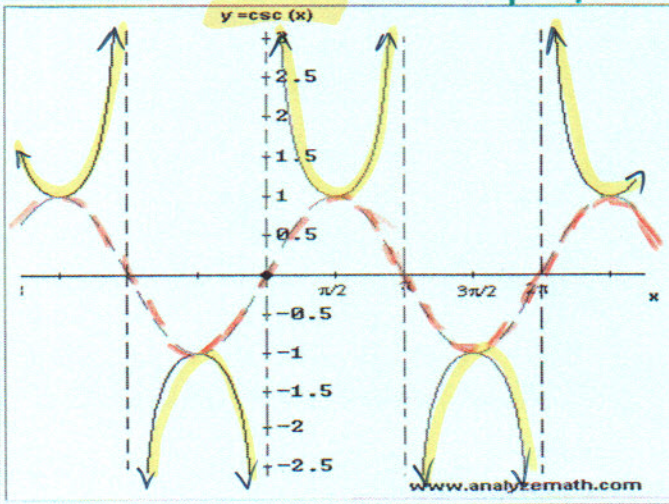
Domain: all real numbers except multiples of π

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$\frac{\sqrt{3}}{2}$	-1	0
$y = \csc x = 1/(\sin x)$	U	2	$\frac{2\sqrt{3}}{3}$	1	U	$\frac{2\sqrt{3}}{3}$	-1	U

Asymptotes Asymptotes



at $\sin\theta$ min $\csc\theta$ max

$\sin\theta$ max $\csc\theta$ min

Secant is like cosecant in that it is reciprocal function. So plot the cosine function, locate vertical asymptotes at $\cos x = 0$, and graph the secant function.

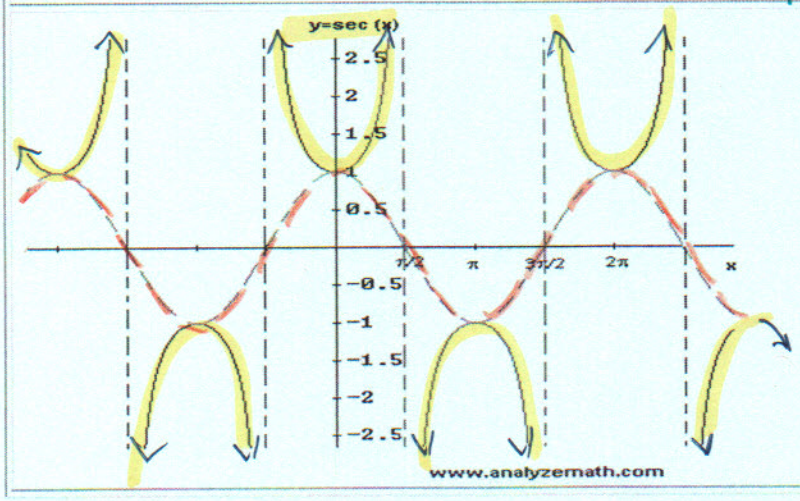
Graph $sec\theta$. First plot $\cos x$.

Where $\cos\theta = 0$
$\sec\theta$ is undefined
∴ Asymptotes

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

input x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	-1	0	1
$y = \sec x$	1	$\frac{2\sqrt{3}}{3}$	2	U	2	-1	U	1

Asymptotes



cosθ min
secθ max

cosθ max
secθ min

note: *where the sine and cosine functions are equal to zero, this is where we see the asymptotes for the reciprocals cosecant and secant. Understand that this will remain the same even when we have a vertical slide.*

$y = A\csc\omega x + B$ $y = A\sec\omega x + B$ $period = \frac{2\pi}{\omega}$

1. graph the reciprocal function of sine or cosine, *the guide functions,*
2. Dash in the guide function.
3. Sketch vertical asymptotes; these occur at the x-values for which the guide function equals 0.
4. now add the vertical slide up or down of **B** units
5. Sketch the typical ∪-shaped branches, approaching the asymptotes that typify the cosecant and secant functions. Note the function's minimum is its reciprocal's maximum.

$$y = 3\sec\frac{1}{2}x$$

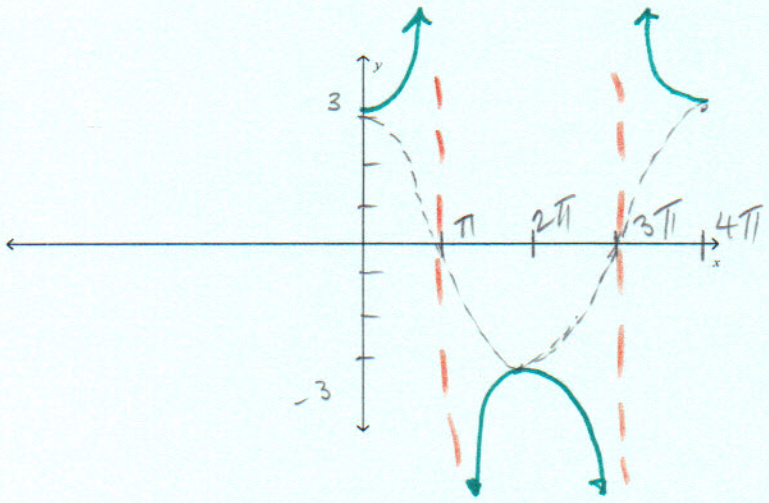
$\omega = \frac{1}{2}$ T= $\frac{2\pi}{\frac{1}{2}} = 4\pi$ A= 3

Interval $[0, 4\pi]$

first graph guide function: $y = 3\cos\frac{1}{2}x$

x	0	π	2π	3π	4π
$\frac{1}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos\frac{1}{2}x$	1	0	−1	0	1
$3\cos\frac{1}{2}x$	3	0	−3	0	3

← Asymptotes



$y = 2\csc x - 1$

$T = 2\pi$ $A = 2$ ↓ 1

$\omega = 1$

first graph "guide function": $y = 2\sin x - 1$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$2\sin x$	0	2	0	-2	0
$2\sin x - 1$	-1	1	-1	-3	-1

← Asymptotes