Homework 14.1: Finding Limits Numerically and Graphically
Place answers on this paper.

#1-3 Complete the table of values (to five decimal places) and use the table to estimate the value of the limit.

1. $\lim_{x\to 2} \frac{x-2}{x^2+x-6} =$ .2

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.20408	.20040	.20004	.19966	.19960	.19608

$\rightarrow \leftarrow$

2. $= \lim_{x\to 0} \frac{e^x-1}{x} =$ 1

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.95163	.99502	.99950	1.00050	1.00510	1.0517

$\rightarrow \leftarrow$

3. $\lim_{x\to 0} \frac{\sin x}{x} =$ 1

x	± 1	± 0.5	± 0.1	± 0.05	± 0.01
$f(x)$					

$\rightarrow \leftarrow$

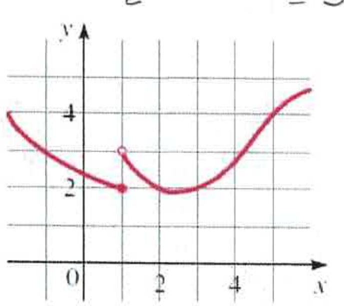
#4-5 Use a table of values to estimate the value of the limit.

4. $\lim_{x\to -4} \frac{x+4}{x^2+7x+12} = -1$

5. $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x} = \frac{1}{6}$

6. For the function f whose graph is given, state the value of the given quantity, if it exists.

a) $\lim_{x\to 1^-} f(x) = 2$ b) $\lim_{x\to 1^+} f(x) = 3$ c) $\lim_{x\to 1} f(x)$ DNE d) $\lim_{x\to 5} f(x) = 4$ e) $f(5) = 4$



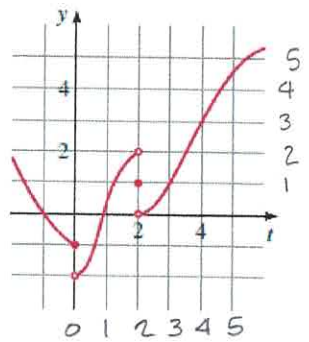
14.1 p2/2

7. For the function g whose graph is given, state the value of the given quantity, if it exists.

$a)\lim_{x\to 0^-} g(t) = -1$ $\quad b)\lim_{x\to 0^+} g(t) = -2$ $\quad c)\lim_{x\to 0} g(t) = $ DNE

$d)\lim_{x\to 2^-} g(t) = 2$ $\quad e)\lim_{x\to 2^+} g(t) = 0$ $\quad f)\lim_{x\to 2} g(t) = $ DNE

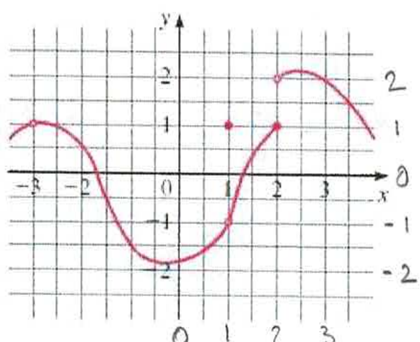
$g)g(2) = 1$ $\quad h)\lim_{x\to 4} g(t) = 3$



8. State the value of the limit, if it exists, from the given graph of f.

$a)\lim_{x\to 3} f(x) = 2$ $\quad b)\lim_{x\to 1} f(x) = -1$ $\quad c)\lim_{x\to -3} f(x) = 1$

$d)\lim_{x\to 2^-} f(x) = 1$ $\quad e)\lim_{x\to 2^+} f(x) = 2$ $\quad f)\lim_{x\to 2} f(x) = $ DNE

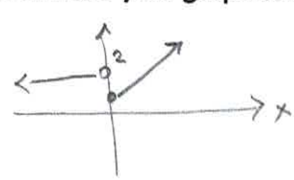


Grid on graph! careful!

#9-10 Graph the piecewise-defined function and use your graph to find the values of the limits, if they exist.

9.

$$f(x)=\begin{cases}2 & \text{if } x<0\\ x+1 & \text{if } x\ge 0\end{cases}$$

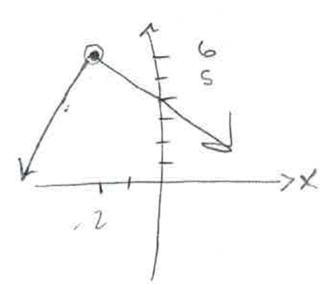


$a)\lim_{x\to 0^-} f(x) = 2$ $\quad b)\lim_{x\to 0^+} f(x) = 1$ $\quad c)\lim_{x\to 0} f(x) = $ DNE

10.

$$f(x)=\begin{cases}2x+10 & \text{if } x\le -2\\ -x+4 & \text{if } x>-2\end{cases}$$

(a) $\lim_{x\to -2} f(x)$ $= 6$ $\quad$ (b) $\lim_{x\to -2^-} f(x)$ $= 6$ $\quad$ (c) $\lim_{x\to -2} f(x)$ $= 6$



Homework 14.2: Finding Limits Algebraically
For credit, please show all work on separate paper.

1. Suppose that

$$\lim_{x\to a} f(x) = -3 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 8$$

Find the value of the given limit, if it exists.

$a)\ \lim_{x\to a}\left[f(x)+h(x)\right] = 5$ $\qquad b)\ \lim_{x\to a}\left[f(x)\right]^3 = -27$

$c)\ \lim_{x\to a}\sqrt[3]{h(x)} = 2$ $\qquad d)\ \lim_{x\to a}\frac{1}{f(x)} = -1/3$

$e)\ \lim_{x\to a}\frac{f(x)}{h(x)} = -3/8$ $\qquad f)\ \lim_{x\to a}\frac{g(x)}{f(x)} = 0$

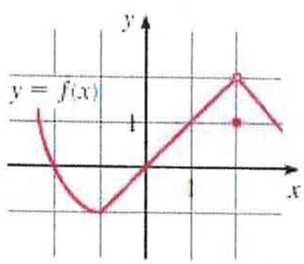
$g)\ \lim_{x\to a}\frac{f(x)}{g(x)} = \text{DNE}$ $\qquad h)\ \lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} = \frac{-6}{11}$

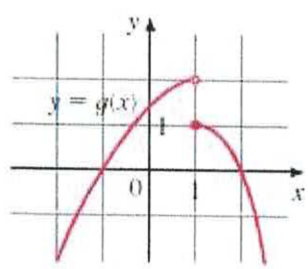
2. The graphs of f and g are given. Use them to evaluate each limit, if it exists.

(a) $\lim_{x\to 2}[f(x)+g(x)] = 2$ $\qquad$ (b) $\lim_{x\to 1}[f(x)+g(x)] = \text{DNE}$

(c) $\lim_{x\to 0}[f(x)g(x)] = 0$ $\qquad$ (d) $\lim_{x\to -1}\frac{f(x)}{g(x)} = \text{DNE}$

(e) $\lim_{x\to 2}x^3 f(x) = 16$ $\qquad$ (f) $\lim_{x\to 1}\sqrt{3+f(x)} = 2$





#3-14 Evaluate the limit, if it exists.

3. $\lim_{x\to 4}\left(5x^2-2x+3\right) = 75$

4. $\lim_{x\to -1}\frac{x-2}{x^2+4x-3} = \frac{1}{2}$

5. $\lim_{x\to 1}\left(\frac{x^4+x^2-6}{x^4+2x+3}\right)^2 = \frac{4}{9}$

6. $\lim_{u\to -2}\sqrt{u^4+3u+6} = 4$

7. $\lim_{x\to 2}\frac{x^2+x-6}{x-2} = 5$

factor

8. $\lim_{x\to 1}\frac{x^3-1}{x^2-1} = \frac{3}{2}$

Expand

9. $\lim_{h\to 0}\frac{(2+h)^3-8}{h} = 12$

factor

10. $\lim_{x\to 2}\frac{x^4-16}{x-2} = 32$

11. $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h} = -\frac{1}{9}$

Neg. exp - rewrite, common denominator, simplify

-common denom.

12. $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \frac{-1}{16}$

factor

13. $\lim_{x\to -1}\frac{x^2-x-2}{x^3-x} = -3/2$

factor

14. $\lim_{x\to 1}\frac{x^8-1}{x^5-x} = 2$

Homework 14.3: Tangent Lines and Derivatives

For credit, please show all work and answers on separate paper

#1-3 Find the slope of the tangent line to the graph of f at the given point.

1. $f(x) = 3x + 4$ $at\ (1, 7)$ $m = 3$
2. $f(x) = 5 - 2x$ $at\ (-3, 11)$ $m = -2$
3. $f(x) = \frac{6}{x+1}$ $at\ (2, 2)$ $m = \frac{-2}{3}$

#4-6 Find an equation of the tangent line to the curve at the given point. Graph the curve and the tangent line. $y = mx + b$

4. $y = 2x - x^3$ $at\ (1, 1)$ $y = -x + 2$
5. $y = \sqrt{x+3}$ $at\ (1, 2)$ $y = \frac{1}{4}x + \frac{7}{4}$
6. $y = \sqrt{1 + 2x}$ $at\ (4, 3)$ $y = \frac{1}{3}x + \frac{5}{3}$

#7-9 Find the derivative of the function at the given number.

7. $f(x) = 2 - 3x + x^2$ $at - 1$ $f' = -4$
8. $g(x) = 2x^2 + x^3$ $at\ 1$ $g' = 7$
9. $F(x) = \frac{1}{\sqrt{x}}$ $at\ 4$ $F' = -\frac{1}{16}$

#10-12 Find $f'(a)$, where a is in the domain of f.

10. $f(x) = x^2 + 2x$ $f'(a) = 2a + 2$
11. $f(x) = -\frac{1}{x^2}$ $f'(a) = \frac{2}{a^3}$
12. $f(x) = \frac{x}{x+1}$ $f'(a) = \frac{1}{(x+1)^2}$

13. **Velocity of a Ball** If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$. -24 ft/sec

14. **Velocity on the Moon** If an arrow is shot upward on the moon with a velocity of 58 m/s, its height (in meters) after t seconds is given by $H = 58t - 0.83t^2$
    a. Find the velocity of the arrow after one second. 56.34 m/sec
    b. Find the velocity of the arrow when $t = a$. $58 - 1.66a$ m/sec
    c. At what time t will the arrow hit the moon? 69.88 sec
    d. With what velocity will the arrow hit the moon? -58 m/sec

15. **Inflating a Balloon** A spherical balloon is being inflated. Find the rate of change of the surface area ($S = 4\pi r^2$) with respect to the radius r when $r = 2\ ft$.

$16\pi\ ft^2/t$

Homework 14.4: Limits at Infinity; Limits of Sequences
For credit, please show work and answers on separate paper

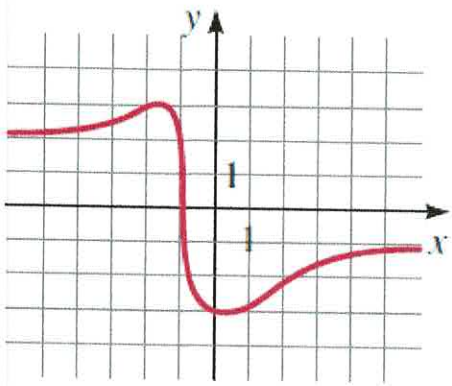
#1-2 a) Use the graph of f to find the following limts.

i) $\lim\limits_{x\to\infty} f(x)$

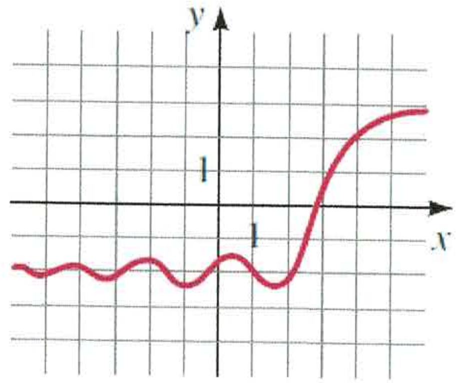
ii) $\lim\limits_{x\to-\infty} f(x)$

b) State the equations of the horizontal asymptotes.

1. i) = -1   ii) = 2



2. i) = -   ii)



#3-7 Find the limit.

3. $\lim\limits_{x\to\infty} \dfrac{3}{x^4}$ = 0

4. $\lim\limits_{x\to\infty} \dfrac{2x+1}{5x-1}$ = 2/5

5. $\lim\limits_{x\to\infty} \dfrac{4x^2+1}{2+3x^2}$ = 4/3

6. $\lim\limits_{x\to\infty} \dfrac{x^4}{1-x^2+x^3}$ = DNE

7. $\lim\limits_{x\to\infty} \cos x$ = DNE

#8-9 Use graphing calculator to estimate limit.

8. $\lim\limits_{x\to\infty} \left(\sqrt{9x^2+x}-3x\right)$ = 1/6

9. $\lim\limits_{x\to\infty} \dfrac{x^5}{e^x}$ = 0

#10-14 Determine if the sequence is convergent or divergent, if it is convergent, find its limit.

10. $a_n = \dfrac{1+n}{n+n^2}$ 0

11. $a_n = \dfrac{n^2}{n+1}$ Divergent

12. $a_n = \cos n\pi$ Divergent

13. $a_n = \dfrac{24}{n^3}\left[\dfrac{n(n+1)(2n+1)}{6}\right]$ = 8

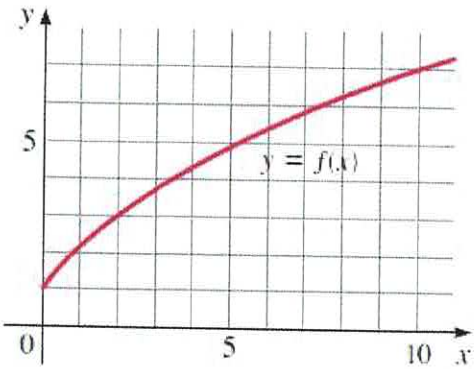
14. $a_n = \dfrac{12}{n^4}\left[\dfrac{n(n+1)}{2}\right]^2$ = 3

Lesson 14.5: The Area Problem; The Integral

For credit, please show all work and answer on separate paper.

1. a) By reading values from the given graph of f, use five rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$ In each case, sketch the rectangles that you use.

b) Find new estimates using ten rectangles in each case



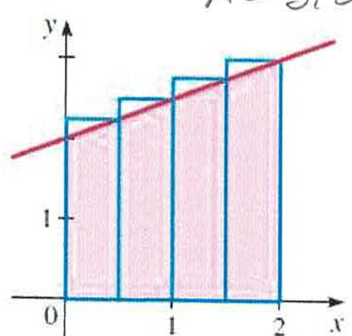
a) $40 \le A \le 52$

b) $42.6 \le A \le 49.1$

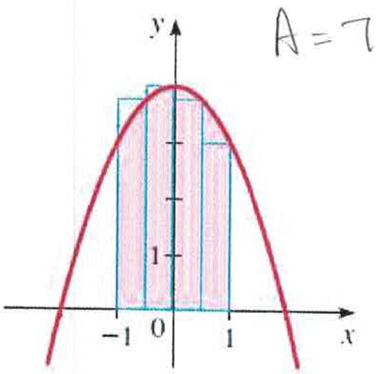
Your estimates may be slightly different, but should be close.

#2-4 Approximate the area of the shaded region under the graph of the given function by using the indicated rectangles. (The rectangles have equal length.)

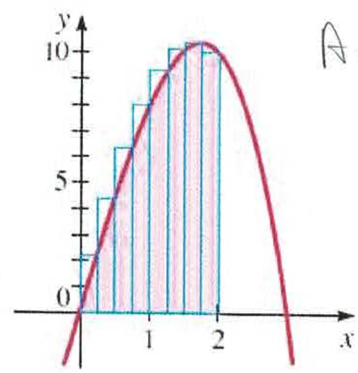
2. $f(x) = \frac{1}{2}x + 2$ $A = 5.25$



3. $f(x) = 4 - x^2$ $A = 7.25$



4. $f(x) = 9x - x^3$



$A = 15.1875$
$= \frac{243}{16}$

5. Use the definition of area as a limit to find the area of the region that lies under the curve.
$y = 3x,\ 0 \le x \le 5$ $A = 75/2$

#6-8 Find the area of the region that lies under the graph of f over the given interval.

6. $f(x) = 3x^2,\ 0 \le x \le 2$ $A = 8$

7. $f(x) = x + x^2,\ 0 \le x \le 1$ $A = 5/6$

8. $f(x) = 20 - 2x^2,\ 2 \le x \le 3$ $A = \frac{22}{3}$