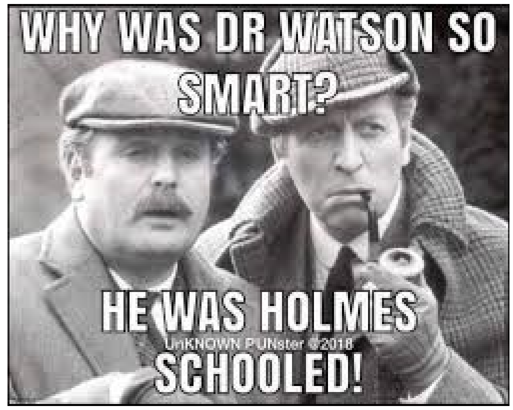
Precalculus
Lesson 12.4: Mathematical Induction
Mrs. Snow, Instructor



Mathematical induction is a method for proving that statements involving natural numbers are true for all natural numbers.

The Principle of Mathematical Induction

Suppose that the following two conditions are satisfied with regard to a statement about natural numbers:

CONDITION I: The statement is true for the natural number 1.

CONDITION II: If the statement is true for some natural number k, it is also true for the next natural number $k + 1$.

Then the statement is true for all natural numbers.

translation:
*#1 show statement is true for **n=1***
*#2 assume statement is true for **n=k**,*
*now show statement is true for **n=k+1** ∴ true for all numbers*

Show that the following statement is true for all natural numbers n.

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

$a_n = 2n - 1$
$S_n = n^2$

#1 show true for $n = 1$

$2(1) - 1 = 1^2$
$2 - 1 = 1$
$1 = 1$ ✓

#2 true for $n = k$:

$1 + 3 + 5 + \cdots + (2k - 1) = k^2$

$a_{k+1} = 2(k+1) - 1$
$S_{k+1} = (k+1)^2$

show true for $n = k+1$

equal ↙ substitute

$1 + 3 + 5 + \cdots (2k - 1) + (2(k+1) - 1) = (k+1)^2$

$k^2 + (2(k+1) - 1) = (k+1)^2$

$k^2 + 2k + 2 - 1 =$

$k^2 + 2k + 1 =$

$(k+1)(k+1) =$

$(k+1)^2 =$ RHS

QED ☺

Show that the following statement is true for all natural numbers n.

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Press pause on the video and work this problem out, then hit play when ready to continue.

Show that the following statement is true for all natural numbers n.

$$\boxed{1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}}$$

$a_n = n$

$S_n = \frac{n(n+1)}{2}$

#1 show true for $n = 1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = \frac{2}{2} \checkmark$$

#2 true for $n = k$

$$1 + 2 + 3 + \ldots\ldots K = \frac{K(K+1)}{2}$$

Show true for $n = k+1$

equal

$$1 + 2 + 3 + \ldots K + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{K(k+1)}{2} + k+1 = \frac{(k+1)(k+2)}{2}$$

$$\frac{K^2 + k}{2} + \frac{(k+1)2}{2} =$$

$$\frac{K^2 + k + 2k + 2}{2} =$$

$$\frac{k^2 + 3k + 2}{2} =$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2} \text{ RHS}$$

QED ☺

∴ true for all numbers

Show that the following statement is true for all natural numbers n.

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$$

Press pause on the video and work this problem out, then hit play when ready to continue.

Show that the following statement is true for all natural numbers n.

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$$

#1 true for $n = 1$ $\quad 3(1) - 2 = \frac{1}{2}(1)(3(1) - 1)$

$3 - 2 = \frac{1}{2}(2)$

$1 = 1$ ✓

#2 true for $n = k$

$1 + 4 + 7 + \ldots + (3k - 2) = \frac{1}{2}k(3k - 1)$

Show true for $n = k + 1$ equal

$1 + 4 + 7 + \ldots (3k - 2) + (3(k+1) - 2) = \frac{1}{2}(k+1)(3(k+1) - 1)$

$\frac{1}{2}k(3k - 1) + (3(k+1) - 2) = \frac{1}{2}(k+1)(3(k+1) - 1)$

$\frac{3k^2 - k}{2} + 3k + 3 - 2 = \frac{1}{2}(k+1)(3k + 3 - 1)$

$\frac{3k^2 - k}{2} + (3k + 1)\left(\frac{2}{2}\right) = \frac{1}{2}(k+1)(3k + 2)$

$\frac{3k^2 - k + 6k + 2}{2} = \quad = \frac{(k+1)(3k+2)}{2}$

$\frac{3k^2 + 5k + 2}{2} =$

$\frac{(3k + 2)(k + 1)}{2} =$ RHS

QED