Precalculus
Lesson 14.1: Finding Limits Using Tables and Graphs
Mrs. Snow, Instructor

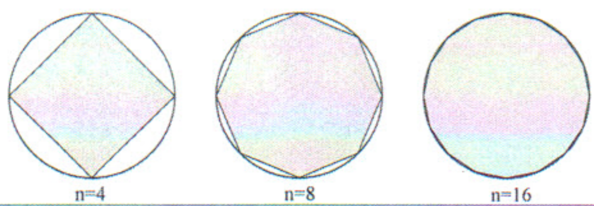
I Like Pushing Things to the Limits

$$\frac{d}{dx}f(x) = \lim_{\Delta \to 0} \frac{f(x+\Delta) - f(x)}{\Delta}$$

ClassicGeek.com

The title of this chapter is "Limits: A Preview of Calculus." The central idea behind calculus is the concept of a **limit.** Calculus is used in modeling numerous real-life phenomena, particularly situations that involve change or motion. To better understand limits let's look back at the Greeks some 2500 years ago and how they used the "method of exhaustion" to find areas. To find the area of a circle for example, the Greeks inscribed a polygon inside the curved region. As the number of sides of the polygon increases, the polygon's area approaches the area of the circle. In other words:

$$area = \lim_{n \to \infty} a_n$$



Definition of a Limit:

We write:

$$\lim_{x \to a} f(x) = L$$

We say: The limit of f(x), as x approaches a, equals L.

We mean: as x gets closer and closer to a, the y value gets closer and closer to L.

Finding a Limit from a Table

Find the limit of

$$\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2} = \frac{1}{6}$$

FROM THE RIGHT

t	0.5	0.1	0.01	0.001
f(x)	.16553	.16662	.16667	.16667

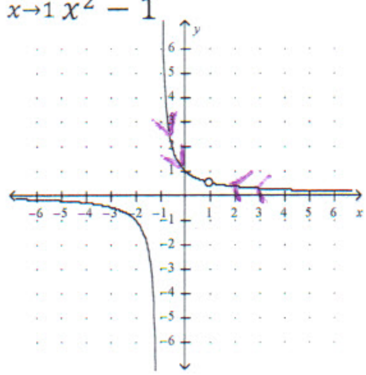
FROM THE LEFT

Both approach $= \frac{1}{6}$

t	−0.5	−0.1	−0.01	−0.001
f(x)	.16553	.16662	.16667	.16667

Find the limit from a table and estimate using the graph of (what are the restrictions?):

$$\lim_{x\to 1}\frac{x-1}{x^2-1}$$



x	f(x)
1.1	.48
1.01	.497
1.001	.5
.9	.526
.99	.50

Also $f(x) = \frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

a hole at $x=1$

for $\frac{1}{x+1}$ $x=1$

$y=\frac{1}{2}$

As graph approaches $x=1$

$y=?$

$y=\frac{1}{2}$

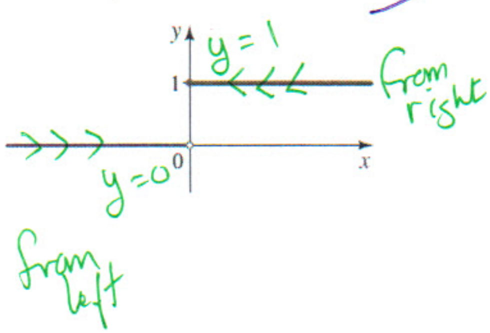
Limits That Fail to Exist: A Function With a Jump

$$H(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t\geq 0 \end{cases}$$

DNE

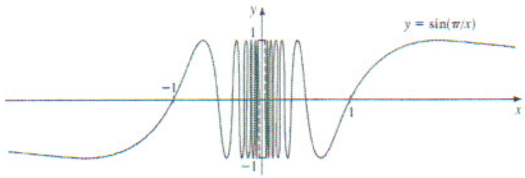
Limit does not exist

limit from the left is different from limit from right.



Limits That Fail to Exist: A Function That Oscillates

Find $\lim_{x\to 0} \sin\frac{\pi}{x}$ = DNE



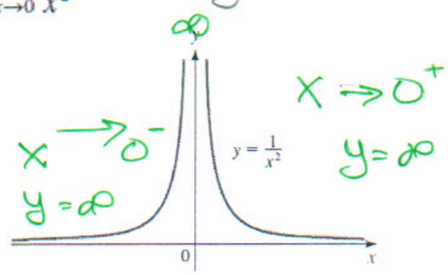
x	f(x)
.06	.866
.006	.866
-.06	-.866
-.006	-.866

↑ going crazy at origen

oscillates → no set value

Limits That Fail to Exist: A Function with a Vertical Asymptote

$\lim_{x\to 0} \frac{1}{x^2}$ → goes to ∞ ∴ In precalculus we say DNE

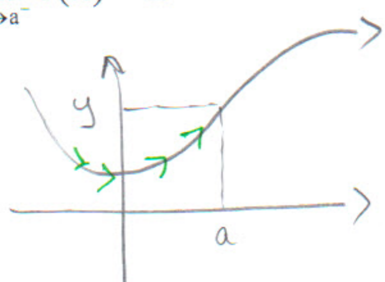


as x gets closer to the value of 0, y explodes going to infinity

One Sided Limits

Left Sided Limit

$\lim_{x\to a^-} f(x) = L$

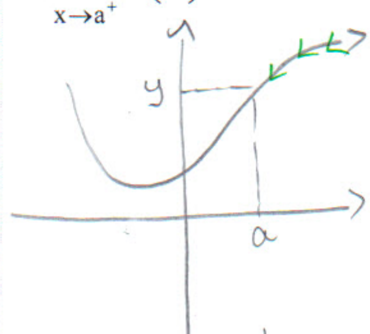


$x \to a^-$

as x approaches a from the negative

Right Sided Limit

$\lim_{x\to a^+} f(x) = L$



$x \to a^+$

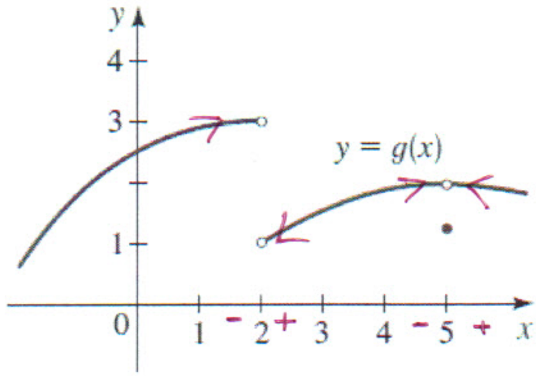
as x approaches a from the positive

Very Important!!!!

$$\lim_{x\to a} f(x) = L \quad \text{if and only if} \quad \lim_{x\to a^-} f(x) = L \quad \text{AND} \quad \lim_{x\to a^+} f(x) = L$$

BOTH LIMITS MUST HAVE SAME VALUE!

Limits From a Graph



$\lim_{x\to 2^-} g(x) = -3$

$\lim_{x\to 2^+} g(x) = 1$

Different

$\lim_{x\to 2} g(x) =$ DNE

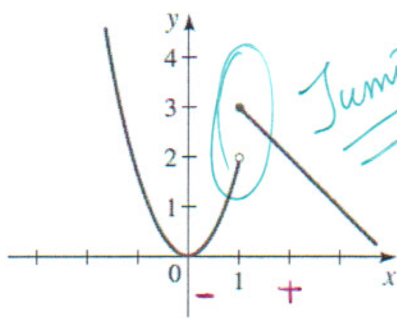
$\lim_{x\to 5^-} g(x) = 2$

$\lim_{x\to 5^+} g(x) = 2$

SAME

$\lim_{x\to 5} g(x) = 2$

A Piecewise-Defined Function



Jump

$$f(x) = \begin{cases} 2x^2 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$

(a) $\lim_{x\to 1^-} f(x) = 2$ (b) $\lim_{x\to 1^+} f(x) = 3$ (c) $\lim_{x\to 1} f(x) =$ DNE