Review 5.1-5.4

5.1

47. $y = (\ln x)^4$

$y' = 4(\ln x)^3\left(\frac{1}{x}\right)$

$= \boxed{\frac{4(\ln x)^3}{x}}$

53. $g(t) = \frac{\ln t}{t^2}$

$g'(t) = \frac{\frac{1}{t}t^2 - 2t\ln t}{t^4}$

$= \frac{t(1-2\ln t)}{t^4}$

$= \boxed{\frac{1-2\ln t}{t^3}}$

59. $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

$= \frac{1}{2}\ln(4+x^2) - \ln x$

$f' = \frac{1}{2}\left(\frac{1}{4+x^2}\right)2x - \frac{1}{x}$

$= \frac{x}{4+x^2} - \frac{1}{x} =$

$\frac{x^2 - 4 - x^2}{(4+x^2)(x)} = \boxed{\frac{-4}{(4+x^2)(x)}}$

71. $f(x) = 3x^2 - \ln x \quad (1,3)$

$f' = 6x - \frac{1}{x} =$

$6(1) - 1 = \underline{5 = m}$

$y = mx+b: \; 3 = 5(1) + b$

$\underline{-2 = b}$

$\boxed{y = 5x - 2}$

86. $y = \frac{\ln x}{x}$

$y' = \frac{\frac{1}{x}(x) - \ln x(1)}{x^2} = \frac{1-\ln x}{x^2}$

$1 - \ln x = 0$

$\ln x = 1$

$\log_e x = 1$

$e^1 = x$

$y = \frac{\ln e}{e} = \frac{1}{e}$

	2	3
	+	−
	/\	max

$\boxed{\max\left(e, \frac{1}{e}\right)}$

97. $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}$

$\ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$

$\frac{y'}{y} = \frac{1}{x} + \frac{3}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\left(\frac{1}{x+1}\right)$

$\frac{y'}{y} = \frac{2(x+1)(x-1) + 3(x)(x+1) - x(x-1)}{2(x)(x-1)(x+1)} =$

$\frac{y'}{y} = \frac{2x^2 - 2 + 3x^2 + 3x - x^2 + x}{2(x)(x-1)(x+1)} =$

$y' = \frac{4x^2+4x-2}{2(x)(x-1)(x+1)}\left(\frac{x(x-1)^{3/2}}{(x+1)^{1/2}}\right)$

$= \frac{2(2x^2+2x-1)(x-1)^{1/2}}{2(x+1)^{3/2}}$

$= \boxed{\frac{(2x^2+2x-1)(x-1)^{1/2}}{(x+1)^{3/2}}}$

5.2

5. $\int \frac{1}{3-2x}\,dx$ $\quad u = 3-2x$, $du = -2dx$, $-\frac{1}{2}du = dx$

$-\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u| + C = \boxed{-\frac{1}{2}\ln|3-2x| + C}$

11. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$ $\quad u = x^3+3x^2+9x$, $du = 3x^2+6x+9\,dx$, $du = 3(x^2+2x+3)dx$, $\frac{1}{3}du = (x^2+2x+3)dx$

$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + C}$

23. $\int \frac{2x}{(x-1)^2}\,dx$ $\quad u = x-1$, $u+1 = x$, $du = dx$

$= 2\int \frac{u+1}{u^2}\,du$

$= 2\int \frac{u}{u^2} + \frac{1}{u^2}\,du = 2\int \frac{1}{u} + u^{-2}\,du$ $\quad (+1)$

$= 2\left(\ln|u| - \frac{1}{u} + C\right) =$

$\boxed{2\ln|x-1| - \frac{2}{x-1} + C}$

25. $\int \frac{1}{1+\sqrt{2x}}\,dx$ $\quad u = 1+(2x)^{1/2}$, $du = \frac{1}{2}(2x)^{-1/2}(2)\,dx$, $du = (2x)^{-1/2}\,dx$, $(2x)^{1/2}du = dx$, $u-1 = (2x)^{1/2}$, $(u-1)du = dx$

$= \int \frac{u-1}{u}\,du$

$= \int 1 - \frac{1}{u}\,du$

$= u - \ln|u| + C$

$= \boxed{1+\sqrt{2x} - \ln|1+\sqrt{2x}| + C}$

31. $\int \csc 2x\,dx =$ $\quad u = 2x$, $du = 2dx$, $\frac{1}{2}du = dx$

$= -\frac{1}{2}\ln|\csc u + \cot u| + C$

$= \boxed{-\frac{1}{2}\ln|\csc 2x + \cot 2x| + C}$

49. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$ $\quad u = 1+\ln x$, $du = \frac{1}{x}dx$

$\int u^2\,du = \frac{u^3}{3}$

$= \frac{(1+\ln x)^3}{3}\Big|_1^e$

$= \frac{(1+\ln e)^3}{3} - \frac{(1+\ln 1)^3}{3}$ $\quad (\ln e = 1,\ \ln 1 = 0)$

$\frac{2^3}{3} - \frac{1}{3} = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

5.4

35. $f(x) = e^{2x}$

$f' = \boxed{2e^{2x}}$

41. $y = \ln(1+e^{2x})$

$y' = \frac{1}{1+e^{2x}} \cdot e^{2x} \cdot 2$

$y' = \boxed{\frac{2e^{2x}}{1+e^{2x}}}$

5.2 #89 see last page.

Review 5.1-5.4

53. $y = x^2e^x - 2xe^x + 2e^x \quad (1, e)$

$y' = 2xe^x + x^2e^x - 2e^x - 2xe^x + 2e^x$

$y' = x^2e^x$

$= (1^2)(e^1) = e = m$

$e = e(1) + b$

$b = 0 \rightarrow \boxed{y = ex}$

58. $e^{xy} + x^2 - y^2 = 10$

$e^{xy}\left(y + x\frac{dy}{dx}\right) + 2x - 2y\frac{dy}{dx} = 0$

$ye^{xy} + xe^{xy}\frac{dy}{dx} + 2x - 2y\frac{dy}{dx} = 0$

$\left(xe^{xy} - 2y\right)\frac{dy}{dx} = -2x - ye^{xy}$

$\frac{dy}{dx} = \boxed{\frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$

85. $\int e^{5x}(5)\,dx = \quad u = 5x, \; du = 5\,dx$

$\int e^u\,du = e^u + C$

$= \boxed{e^{5x} + C}$

89. $\int \frac{e^{-x}}{1 + e^{-x}}\,dx \quad u = 1 + e^{-x}, \; du = -e^{-x}\,dx, \; -du = e^{-x}\,dx$

$= -\int \frac{1}{u}\,du = -\ln|u|$

$= \boxed{-\ln|1 + e^{-x}| + C}$

91. $\int e^x\sqrt{1 - e^x}\,dx \quad u = 1 - e^x, \; du = -e^x\,dx, \; -du = e^x\,dx$

$= \int -u^{1/2}\,du =$

$-\frac{2}{3}u^{3/2} =$

$\boxed{-\frac{2}{3}(1 - e^x)^{3/2} + C}$

97. $\int e^{-x}\tan e^{-x}\,dx \quad u = e^{-x}, \; du = -e^{-x}\,dx$

$= -\int \tan u\,du$

$= \ln|\cos u| = \boxed{\ln|\cos e^{-x}| + C}$

103. $\int_1^3 \frac{e^{3/x}}{x^2}\,dx \quad u = \frac{3}{x} = 3x^{-1}, \; du = -\frac{3}{x^2}\,dx, \; -\frac{1}{3}\,du = \frac{1}{x^2}\,dx$

$= \int -\frac{1}{3}e^u\,du$

$= -\frac{1}{3}e^u = -\frac{1}{3}e^{3/x}\Big|_1^3$

$= -\frac{1}{3}e^{3/3} - \left(-\frac{1}{3}e^{3/1}\right) = \boxed{\frac{-e + e^3}{3}}$

106. $\int_{\pi/3}^{\pi/2} e^{\sec 2x}\sec 2x\tan 2x\,dx \quad u = \sec 2x, \; du = 2\sec x\tan x\,dx, \; \frac{1}{2}du = \sec x\tan x\,dx$

$= \int_2^1 \frac{1}{2}e^u\,du = e^u$

$= \frac{1}{2}e^{\sec 2x}\Big|_{\pi/3}^{\pi/2}$

$= \frac{1}{2}e^{\sec \pi} - \frac{1}{2}e^{\sec 2\pi/3}$

$= \frac{1}{2}e^{-1} - \frac{1}{2}e^{-2}$

$= \frac{1}{e}\frac{1}{2e} - \frac{1}{2e^2} = \boxed{\frac{e - 1}{2e^2}}$

5.1 - 5.4 Review

5.2

89

$$f(x) = \frac{\ln x}{x} \qquad [1, e]$$

$$\frac{1}{e-1}\int_1^e \frac{\ln x}{x}\,dx \qquad u = \ln x \qquad du = \frac{1}{x}\,dx$$

$$\frac{1}{e-1}\int_1^e u\,du =$$

$$\frac{1}{e-1}\left(\frac{u^2}{2}\right) \Rightarrow \frac{1}{e-1}\left.\frac{(\ln x)^2}{2}\right|_1^e$$

$$\frac{1}{e-1}\frac{(\ln e)^2}{2} - \frac{1}{e-1}\frac{(\ln 1)^2}{2}$$

$$\left(\frac{1}{e-1}\right)\left(\frac{1}{2}\right) = \boxed{\frac{1}{2e-2}}$$