5.1

P1/3

45. $g(x) = \ln x^2$

$g'(x) = \frac{1}{x\cancel{x}}(2\cancel{x}) = \boxed{\frac{2}{x}}$

48. $y = x \ln x$

$y' = (1)\ln x + \left(\frac{1}{x}\right)x =$

$y' = \boxed{\ln x + 1}$

51. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$= \ln x - \ln(x^2+1)$

$= \frac{1}{x} - \frac{1}{x^2+1}(2x)$

$= \boxed{\frac{1}{x} - \frac{2x}{x^2+1}}$

54. $h(t) = \frac{\ln t}{t}$

$h'(t) = \frac{t\left(\frac{1}{t}\right) - (1)(\ln t)}{t^2}$

$= \boxed{\frac{1 - \ln t}{t^2}}$

57. $y = \ln\sqrt{\frac{x+1}{x-1}} = \ln\left(\frac{x+1}{x-1}\right)^{1/2} =$

$= \frac{1}{2}\left[\left(\ln(x+1) - \ln(x-1)\right)\right]$

$= \frac{1}{2}\left(\frac{1}{x+1}(1) - \frac{1}{x-1}(1)\right)$

$= \frac{1}{2}\left(\frac{(\cancel{x}-1)-(\cancel{x}+1)}{(x+1)(x-1)}\right) = \frac{1}{2}\left[\frac{-2}{(x+1)(x-1)}\right]$

$= \boxed{\frac{-1}{(x+1)(x-1)}}$

60. $f(x) = \ln\left(x + \sqrt{4+x^2}\right)$

$f' = \frac{1}{x+\sqrt{4+x^2}}\left(1 + \frac{1}{\cancel{2}}(4+x^2)^{-\frac{1}{2}}(\cancel{2}x)\right)$

$= \frac{1}{x+\sqrt{4+x^2}}\left(1 + \frac{x}{\sqrt{4+x^2}}\right)$

$= \left(\frac{1}{\cancel{x+\sqrt{4+x^2}}}\right)\left(\frac{\cancel{\sqrt{4+x^2}\quad+x}}{\sqrt{4+x^2}}\right)$

$= \boxed{\frac{1}{\sqrt{4+x^2}}}$

63. $y = \ln|\sin x|$

$y' = \frac{1}{\sin x}(\cos x)$

$= \frac{\cos x}{\sin x} = \boxed{\cot x}$

66. $y = \ln|\sec x + \tan x|$

$y' = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$

$= \frac{1}{\cancel{(\sec x + \tan x)}}(\sec x)(\cancel{\tan x + \sec x})$

$= \boxed{\sec x}$

72. $f(x) = 4 - x^2 - Ln\left(\frac{1}{2}x+1\right)$ (0,4)

$y' = -2x - \frac{1}{\frac{1}{2}x+1}\left(\frac{1}{2}\right)$

$= -2x - \frac{1}{x+2}$

$m = -2(0) - \frac{1}{0+2} = \boxed{\frac{-1}{2} = m}$

$4 = \frac{-1}{2}(0) + b$

$b = 4$ $\boxed{y = \frac{-1}{2}x + 4}$

75. $f(x) = x^3 Lnx$ (1,0)

$f' = 2x^2 Lnx + x^3\left(\frac{1}{x}\right)$

$= 2x^2 Lnx + x^2$

$m = 2(1^2)Ln1 + 1^2 = 1$

$0 = 1(1) + b$

$-1 = b$ $\boxed{y = x - 1}$

77. $x^2 - 3Lny + y^2 = 10$

$2x - 3\frac{1}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\left(-\frac{3}{y} + 2y\right)\frac{dy}{dx} = -2x$

$\left(\frac{-3+2y^2}{y}\right)\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -2x\left(\frac{y}{-3+2y^2}\right)$

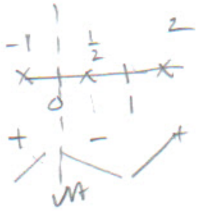
$\frac{dy}{dx} = \boxed{\frac{-2xy}{-3+2y^2}}$

84. $y = x - Lnx$

$y' = 1 - \frac{1}{x} = 0$

$1 = \frac{1}{x}$

$x = 1$



NA at $x = 0$

$\boxed{\text{min } (1, 1)}$ $y = 1 - Ln1$

$y = 1$

87. $y = \frac{x}{Lnx}$

$y' = \frac{Lnx - \frac{1}{x}(x)}{(Lnx)^2} = \frac{Lnx - 1}{(Lnx)^2} = 0$

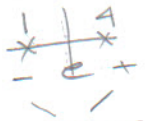
$Lnx = 1$

$Log_e x = 1$

$e^1 = x$

$x = e$

$y = \frac{e}{Lne}$



$\boxed{\text{min } (e, e)}$

93. $y = x\sqrt{x^2-1}$

$\ln y = \ln x\sqrt{x^2-1}$

$\ln y = \ln x + \frac{1}{2}\ln(x^2-1)$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{\cancel{2}}\left(\frac{1}{x^2-1}\right)(\cancel{2}x)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{x}{x^2-1}$$

$$\frac{y'}{y} = \frac{x^2-1+x^2}{x(x^2-1)}$$

$$y' = \frac{2x^2-1}{\cancel{x}(x^2-1)}\left(\cancel{x}(x^2-1)^{\frac{1}{2}}\right)$$

$$= \boxed{\frac{2x^2-1}{(x-1)^{1/2}}}$$

96. $y = \sqrt{\dfrac{x^2-1}{x^2+1}}$

$\ln y = \ln\sqrt{\dfrac{x^2-1}{x^2+1}}$

$\ln y = \frac{1}{2}\left(\ln(x^2-1) - \ln x^2+1\right)$

$$\frac{y'}{y} = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2-1} - \frac{\cancel{2}x}{x^2+1}\right)$$

$$= \frac{\cancel{x(x^2+1)} - \cancel{x(x^2-1)}}{(x^2-1)(x^2+1)}$$

$$\frac{y'}{y} = \frac{2x}{(x^2-1)(x^2+1)}$$

$$y' = \frac{2x}{\cancel{(x^2-1)}^{\frac{1}{2}}(x^2+1)}\;\frac{\cancel{(x^2-1)}^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}$$

$$\boxed{y' = \frac{2x}{(x^2-1)^{1/2}(x^2+1)^{3/2}}}$$

5.2

3. $\int \frac{1}{x+1}\,dx$ $\quad u = x+1$

$du = dx$

$= \int \frac{1}{u}\,du =$

$= \ln|u| + c = \boxed{\ln|x+1| + c}$

6. $\int \frac{1}{3x+2}\,dx =$ $\quad u = 3x+2$

$du = 3dx$

$\frac{1}{3}\int\left(\frac{1}{u}\right)du =$ $\quad \frac{1}{3}du = dx$

$\frac{1}{3}\ln|u| + c = \boxed{\frac{1}{3}\ln|3x+2| + c}$

9. $\int \frac{x^2-4}{x}\,dx = \int\left(\overset{+1}{x} - \frac{4}{x}\right)dx$

$= \boxed{\frac{1}{2}x^2 - 4\ln|x| + c}$

12. $\int \frac{x(x+2)}{x^3+3x^2-4}\,dx = \int \frac{x^2+2x}{x^3+3x^2-4}\,dx$

$u = x^3+3x^2-4$

$du = (3x^2+6x)dx$ $\quad = \frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{3}du = (x^2+2x)dx$ $\quad = \frac{1}{3}\ln|u| + c$

$\boxed{= \frac{1}{3}\ln|x^3+3x^2-4| + c}$

15. $\int \frac{x^3-3x^2+5}{x-3}\,dx$

$$\begin{array}{r|l} & x^2 \\ \hline x-3 & x^3-3x^2+5 \\ & -\,x^3-3x^2 \\ \hline & \quad 5 \end{array}$$

$= \int x^2 + \frac{5}{x-3}\,dx$

$= \boxed{\frac{1}{3}x^3 + 5\ln|x-3| + c}$

18. $\int \frac{x^3-3x^2+4x-9}{x^2+3}\,dx$

$$\begin{array}{r|l} & x-3 \\ \hline x^2+3 & x^3-3x^2+4x-9 \\ & -\,x^3 \qquad\quad +3x \\ \hline & \qquad -3x^2+x-9 \\ & \qquad -3x^2+3x-9 \\ \hline & \qquad\qquad 2x \end{array}$$

$= \int x - 3 + \frac{2x}{x^2+3}\,dx$ $\quad u = x^2+3$

$du = 2x\,dx$

$= \int x - 3 + \left(\frac{1}{2}\right)\frac{1}{u}\,du$ $\quad \frac{1}{2}du = x\,dx$

$= \frac{1}{2}x^2 - 3x + \frac{1}{2}\ln|u| + c$

$\boxed{= \frac{1}{2}x^2 - 3x + \frac{1}{2}\ln|x^2+3| + c}$

21. $\int \frac{1}{\sqrt{x+1}}\,dx =$ $\quad u = x+1$

$du = dx$

$\int \overset{+1}{u^{-1/2}}\,du =$

$2u^{1/2} + c = \boxed{2(x+1)^{1/2} + c}$

24. $\int \frac{x(x-2)}{(x-1)^3}\,dx$ $\quad u = x-1 \quad u+1 = x$

$du = dx$

$= \int \frac{(u+1)(u+1-2)}{u^3}\,du$

$= \int (u+1)(u-1)(u^{-3})\,du$

$= \int (u^2-1)u^{-3}\,du$

$= \int \overset{+1}{u^{-1}} - u^{-3}\,du_{+1}$

$= \int \frac{1}{u} - u^{-3}\,du$

$= \ln|u| - \frac{u^{-2}}{2} + c$

$= \boxed{\ln|x-1| - \frac{1}{2(x-1)^2} + c}$

26. $\int \frac{1}{1+\sqrt{3x}}\,dx =$ $\quad u = 1+\sqrt{3x} \rightarrow u-1=\sqrt{3x}$

$du = \frac{1}{2}(3x)^{-1/2}(3)\,dx$

$\frac{2}{3}(3x)^{1/2}du = dx$

$\frac{2}{3}(u-1)du = dx$

$$= \frac{2}{3}\int \frac{1}{u}\frac{(u-1)}{1}\,du$$

$$= \frac{2}{3}\int \frac{u}{u} - \frac{1}{u}\,du$$

$$\frac{2}{3}\int 1 - \frac{1}{u}\,du = \frac{2}{3}(u - \ln|u|) + c$$

$$\boxed{= \frac{2}{3}\left(1+\sqrt{3x} - \ln|1+3x|\right) + c}$$

27. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$ $\quad u = \sqrt{x}-3 \Rightarrow u+3 = \sqrt{x}$

$du = \frac{1}{2}(x)^{-1/2}dx$

$2(x)^{1/2}du = dx$

$2(u+3)\,du = dx$

$$= \int \frac{(u+3)}{u}(2)(u+3)\,du$$

$$= 2\int \frac{u^2+6u+9}{u}\,du =$$

$$= 2\int (u + 6 + \frac{9}{u}\,du =$$

$$= 2\left(\frac{1}{2}u^2 + 6u + 9\ln|u|\right) + c$$

$$= u^2 + 12u + 18\ln|u| + c$$

$$= (\sqrt{x}-3)^2 + 12(\sqrt{x}-3) + 18\ln|(\sqrt{x}-3)| + c$$

$$x - 6\sqrt{x} + 9 + 12\sqrt{x} - 36 + 18\ln|\sqrt{x}-3| + c$$

$$\boxed{= x + 6\sqrt{x} - 27 + 18\ln|\sqrt{x}-3| + c}$$

30. $\int \tan 5\theta\,d\theta =$ $\quad u = 5\theta$

$du = 5\,d\theta$

$\frac{1}{5}du = d\theta$

$$= \frac{1}{5}\int \tan u\,du$$

$$= -\frac{1}{5}\ln|\cos u| + c$$

$$= \boxed{-\frac{1}{5}\ln|\cos 5\theta| + c}$$

33. $\int \frac{\cos t}{1+\sin t}\,dt$ $\quad u = 1+\sin t$

$du = \cos t\,d$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + c = \boxed{\ln|1+\sin t| + c}$$

36. $\int (\sec t + \tan t)\,dt =$

$$= \ln|\sec t + \tan t| - \ln|\cos t| + c$$

38. $\frac{dy}{dx} = \frac{2x}{x^2-9}$

$u = x^2-9$

$du = 2x\,dx$

$$y = \int \frac{2x}{x^2-9}\,dx$$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + c =$$

$$\boxed{\ln|x^2-9| + c}$$

39. $\frac{ds}{d\theta} = \tan 2\theta$ $\quad u = 2\theta$

$du = 2\,d\theta$

$\frac{1}{2}du = d\theta$

$$s = \int \tan 2\theta\,d\theta$$

$$= \frac{1}{2}\int \tan u\,du$$

$$= \frac{1}{2}\left(-\ln|\cos u| + c\right)$$

$$= -\frac{1}{2}\ln|\cos 2\theta| + c$$

48.

$\int_{-1}^{1} \frac{1}{x+2}\,dx$ $\quad u = x+2$, $du = dx$

$\int_{-1}^{1} \frac{1}{u}\,du = \ln|u|$

$= \ln|x+2|\Big]_{-1}^{1} =$

$\ln|3| - \ln|-1+2|$

$\ln|3| - \cancel{\ln|1|}^{\,0}$

$\boxed{\ln|3|}$

51. $\int_{0}^{2} \frac{x^2-2}{x+1}\,dx$ $\quad$ degree diff = 1 divide

$$\begin{array}{r|l} & x - 1 \\ x+1 & x^2 - 0x - 2 \\ & x^2 + x \\ \hline & -x - 2 \\ & (+x + 1) \\ \hline & -1 \end{array} \rightarrow x - 1 - \frac{1}{x+1}$$

$= \int_{0}^{2} x - 1 - \frac{1}{x+1}\,dx$

$= \frac{1}{2}x^2 - x - \ln|x+1|\Big|_{0}^{2}$

$\frac{1}{2}(4) - 2 - \ln|3| - (0 - 0 - \cancel{\ln 1}^{\,0})$

$\cancel{2-2} - \ln|3|$

$= \boxed{-\ln 3}$

89. $f(x) = \frac{\ln x}{x}$ $\quad [1, e]$

$= \frac{1}{1-e}\int_{1}^{e} \frac{\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$

$\frac{1}{1-e}\int_{1}^{e} u\,du =$

$\frac{1}{1-e}\left(\frac{1}{2}\right)u^2 = \frac{1}{1-e}\left(\frac{1}{2}\right)(\ln x)^2\Big|_{1}^{e}$

$= \frac{1}{1-e}\left[\frac{1}{2}(\ln e)^2 - \frac{1}{2}(\ln 1)^2 =\right.$

$= \frac{1}{1-e}\left[\frac{1}{2}(1)^2 - 0\right]$

$= \frac{1}{1-e}\left(\frac{1}{2}\right) = \boxed{\frac{1}{2-2e}}$