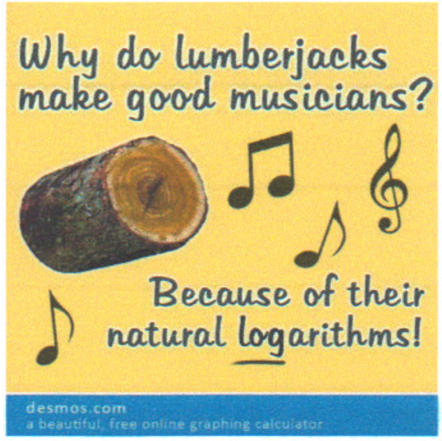
Precalculus

Lesson 5.4: Logarithmic Functions

Mrs. Snow, Instructor



The inverse of an exponential function is a logarithmic function.

Let a be a positive number with a not equal to 1. The logarithmic function with base a is defined by:

The base is the base whether exponent or Log.

$$\log_a x = y$$

if and only if

$$a^y = x$$

log roll – Always start with base

Domain: $(0, \infty)$

translation: whatever you are taking the log of has to be greater than zero

Range: $(-\infty, \infty)$

start with the base and move in a counterclockwise fashion.

Change each logarithmic statement to an equivalent statement involving an exponent.

$\log_a 4 = 5$	$\log_e b = -3$	$\log_3 5 = c$
$a^5 = 4$	$e^{-3} = b$	$3^c = 5$

Change each exponential statement to an equivalent statement involving a logarithm.

$1.2^3 = m$	$e^b = 9$	$a^4 = 24$
Base $\log_{1.2} m = 3$	$\log_e 9 = b$	$\log_a 24 = 4$

Base is in the basement!

Find the exact value.

set = x
log roll

$\log_2 16 = x = ?$	$\log_3 \frac{1}{27} = x$
$2^x = 16$ $2^x = 2^4$ $x = 4$	$3^x = \frac{1}{27}$ $3^x = \frac{1}{3^3}$ $3^x = 3^{-3}$ $\boxed{x = -3}$ opposite side opposite sign

Find the domain of each logarithmic function.

$f(x) = log_2(x+3)$	$g(x) = log_5\left(\frac{1+x}{1-x}\right)$
Argument > 0 $x+3>0$ $x>-3$ D: $(-3, \infty)$	$\frac{1+x}{1-x} > 0$ D: $(-1, 1)$

	-2	-1	0	1	2
$1+x$	$-$		$+$		$+$
$1-x$	$+$		$+$		$-$
	$-$		$+$		$-$

Graphing Logarithmic Functions

Knowing the general form of the graph of the log function is a short cut for graphing.

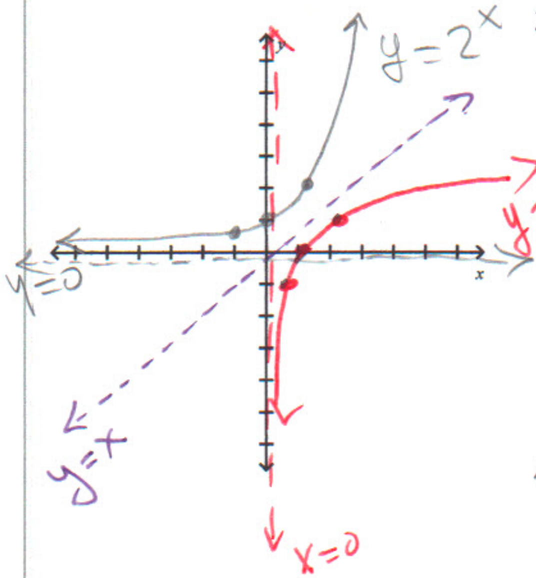
1. Write in its equivalent exponential form
2. Find the inverse; x *is* y *and* y *is* x, solve for y
3. Graph the log function's inverse, and reflect the exponential graph across the line of symmetry $y = x$.

Graph, determine the domain, range and vertical asymptote.

$y = log_2 x \longleftrightarrow 2^y = x$ ①

Switch ②

$2^x = y$ Inverse



$y = 2^x$

x	y
0	1
1	2
-1	$\frac{1}{2}$

Asymptote $y=0$

$\longrightarrow$

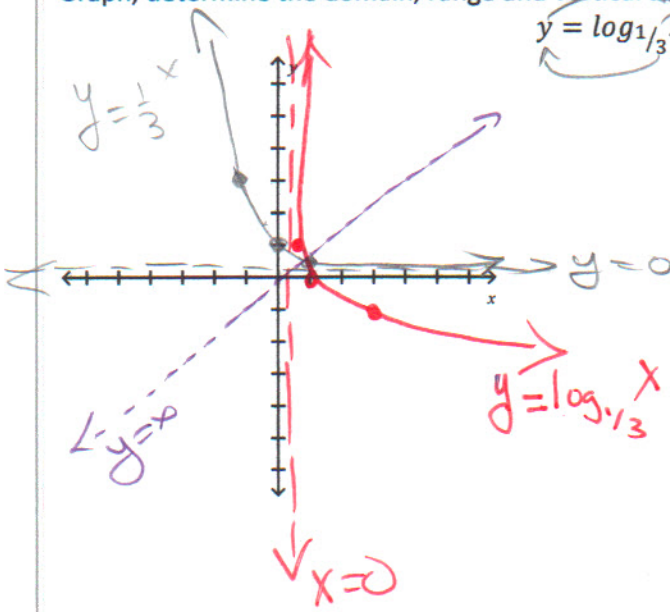
Graph $Log_2 x$

x	y
1	0
2	1
$\frac{1}{2}$	-1

Switch Asymptote $X=0$

Graph, determine the domain, range and vertical asymptote.

$y = log_{1/3} x$ $= \frac{1}{3}^{y} = x$



Inverse

$y = \frac{1}{3}^{x}$

x	y
0	1
1	1/3
-1	3

$y = \log_{1/3} x$

x	y
1	0
1/3	1
3	-1

Switch

Asymptote

$y = 0$ $x = 0$

The Natural and Common Logarithm

The Natural Logarithm is a logarithm with the base e. It is written with the abbreviation of ln.

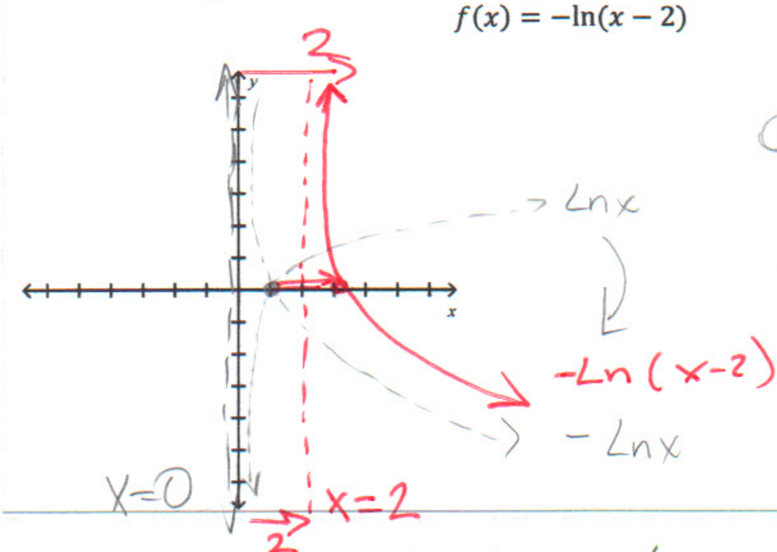
$$y = \ln x, \quad if\ and\ only\ if \quad x = e^y$$

$$y = \log x, \quad if\ and\ only\ if\ x = 10^y$$

The logarithm with base 10 is called the ***common logarithm*** *and is denoted by omitting the base*

Graph, determine the domain, range and vertical asymptote. Identify the inverse and the domain and range of the inverse.

$$f(x) = -\ln(x-2)$$



Parent = ln x
Dash in parent with critical pt.
① reflect over x-axis
② Shift 2 Right →
Both critical pt → 2
& asymptote

Solving Logarithmic Equations We have a log jam → Log roll!

$log_3(4x-7) = 2$ $3^2 = 4x-7$ $9 + 7 = 4x - 7 + 7$ $16 = 4x$ $16/4 = \boxed{4 = x}$	$log_x 64 = 2$ $x^2 = 64$ ← perfect square! $x^2 = 8^2$ $\boxed{x = 8}$

Using Logarithms to Solve and Exponential Equation

$$e^{2x} = 5$$

$$\log_e 5 = 2x$$

$$\frac{\ln 5}{2} = x \approx .805$$

X is in the tree top
Log it!

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Lesson 5.5: Properties of Logarithms
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Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents:

rewrite to see how these properties are true

$$log_a 1 = 0 \qquad a^0 = 1$$

$$log_a a = 1 \qquad a^1 = a$$

$$a^{log_a M} = M \qquad Log_a M = Log_a M$$

$$log_a a^r = r \qquad a^r = a^r$$

Simplify:

Properties are nice but, we have to memorize, so, you can always rewrite

The base is always the base!

$2^{log_2 \pi} = x$

$log_2 x = Log_2 \pi$

$\boxed{x = \pi}$

$log_{0.2} 0.2^{-\sqrt{2}} = x$

$.2^x = .2^{-\sqrt{2}}$

$\boxed{x = -\sqrt{2}}$

$ln e^{kt}$

$log_e e^{kt} = x$

$e^x = e^{kt}$

$\boxed{x = kt}$

$log_4 4 = x$

$4^x = 4^1$

$\boxed{x = 1}$

Properties of Logarithms

Memorize!

$$log_a MN = log_a M + log_a N$$

$$log_a \frac{M}{N} = log_a M - log_a N$$

$$log_a M^r = r\, log_a M$$

Write the logarithmic expressions as Sum and Difference Logs

$$\log_a\left(x\sqrt{x^2+1}\right), x > 0$$

← multiplied → Change Add

$= \log_a x + \log_a \sqrt{x^2+1}$ ← Change to rational exponent & move

$= \log_a x + \log_a (x^2+1)^{1/2}$

$= \boxed{\log_a x + \frac{1}{2}\log_a (x^2+1)}$

$$\ln\frac{x^2}{(x-1)^3}$$ ← divide = Subtract

$= \ln x^2 - \ln(x-1)^3$

Move exponents down

$= \boxed{2\ln x - 3\ln(x-1)}$

$$\log_a\frac{\sqrt{x^2+1}}{x^3(x+1)^4}$$ — divide ⇒ subtract

$= \log_a \sqrt{x^2+1} - [\underbrace{\log_a (x^3)(x+1)^4}_{\text{ALL Denominator}}]$

Be careful!

$= \log_a (x^2+1)^2 - [\log_a x^3 + \log_a (x+1)^4]$

distribute minus

$= \boxed{2\log_a(x^2+1) - 3\log_a x - 4\log_a(x+1)}$

one "Log" term in answer!

$\ln 1 = x$? $\quad \ln 1 = 0$

$\log_e 1 = x$

$e^x = 1 \quad x = 0$

Writing Expressions as a Single Logarithm

$log_a 7 + 4 log_a 3 =$ $\log_a 7 + \log_a 3^4 =$ $\log_a (7)(81) =$ $\boxed{\log_a 567}$	$\frac{2}{3}\ln 8 - \ln(5^2 - 1)$ $\ln 8^{2/3} - \ln(25-1) =$ $\ln 4 - \ln 24 =$ $\ln \frac{4}{24} = \ln \frac{1}{6}$ But.... $\cancel{\ln 1}^{0} - \ln 6 = -\ln 6$ $8^{2/3} = (2^3)^{\frac{2}{3}} = 4$
$log_a x + log_a 9 + log_a (x^2 + 1) - log_a 5$ $\underbrace{}_{\times}$ $\underbrace{}_{\times}$ $\underbrace{}_{\div}$ $\log_a \frac{(x)(9)(x^2+1)}{5} = \log_5 \left(\frac{9x^3+9x}{5}\right)$ And we can simplify	

Change of Base

this allows us to use the calculator using Log base 10 or natural Log

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then
base is in the basement or stays on the bottom $\quad \log_a M = \frac{\log_b M}{\log_b a} \quad$ b is our base 10 or base e
Therefore: $\log_a M = \frac{\log M}{\log a}$ and $\log_a M = \frac{\ln M}{\ln a}$

Using the Change of Base Formula

$log_5 89$ $\log_5 89 = \frac{\log 89}{\log 5} = \frac{\ln 89}{\ln 5}$ $\approx \boxed{2.7889}$	$log_{\sqrt{2}} \sqrt{5}$ $\log_{\sqrt{2}} \sqrt{5} = \frac{\log \sqrt{5}}{\log \sqrt{2}} = \frac{\ln \sqrt{5}}{\ln \sqrt{2}}$ $\approx \boxed{2.3219}$

Need to know this problem too!

Find the exact value of $(\log_2 5)(\log_5 16) = \boxed{4}$

Note: 1st term argument = 2nd term base

Use change of base on 2nd term; Change to Log base 2 $= \left(\cancel{\log_2 5}\right)\left(\frac{\log_2 16}{\cancel{\log_2 5}}\right) =$

$\log_5 16 = \frac{\log_2 16}{\log_2 5}$

$= \log_2 16 = x$

$2^x = 16 \Rightarrow 2^x = 2^4$ so Ans = 4