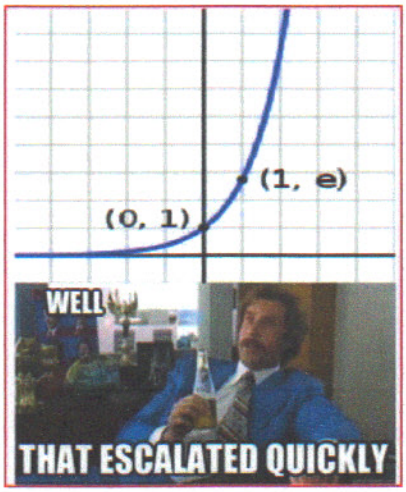
Precalculus

Lesson5.3: Exponential Equations

Mrs. Snow, Instructor

"Exponential"

x is exponent



The exponential function is one of the most important functions in mathematics. The function is used to model the natural process of population growth and radioactive decay. It is also important in finances such interest and depreciation. The exponential function with base ***a*** is defined for all real numbers ***x*** by:

$$f(x) = Ca^x$$

$$y = Ca^x$$

where $a > 0$ *and* $a \neq 1$

a is the growth factor

C is the initial value

when $x = 0$:

$y = Ca^0$

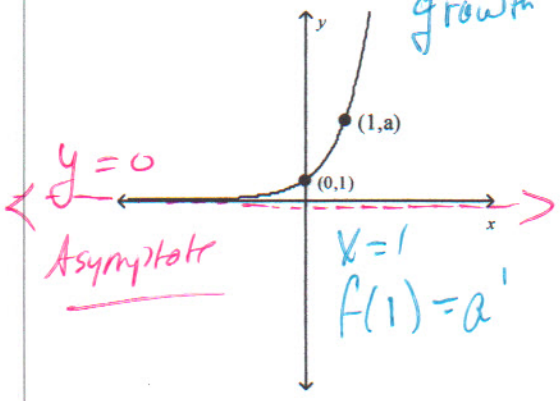
$y = C$

x May be time so at $t = 0$

C = initial value

$f(x) = a^x, \quad a \neq 0$

growth



$y = 0$

Asymptote

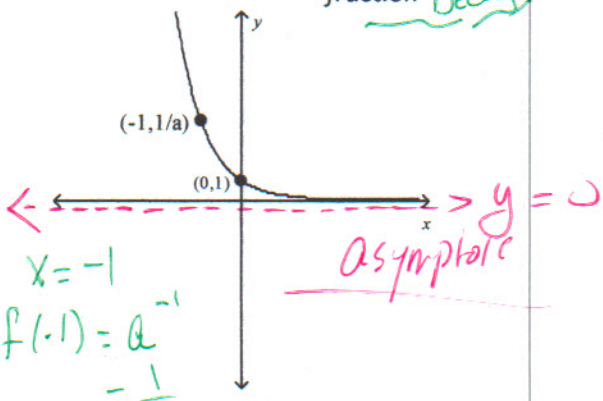
$x = 1$

$f(1) = a^1$

Growth

$f(x) = a^x, \quad 0 < a < 1$

fraction Decay



$y = 0$

asymptote

$x = -1$

$f(-1) = a^{-1}$

$= \frac{1}{a}$

Decay

pass through a common point of (0,1) ⇐(0,1)

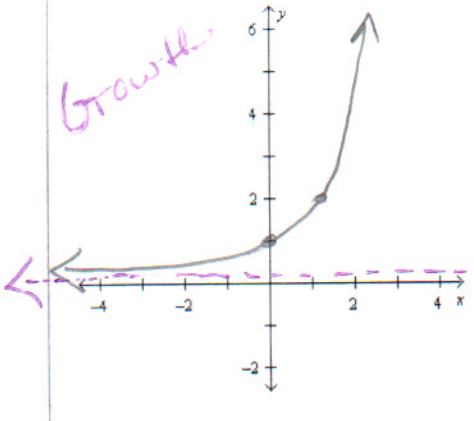
domain= $(-\infty, \infty)$

range $(0, \infty)$

asymptote at $y = 0$

Graph the Exponential Functions

$f(x) = (2)^x$



Growth

$x=0 \quad y=1$

$x=1 \quad y=2$

$y=0$

Graph requirements:

① Asymptote

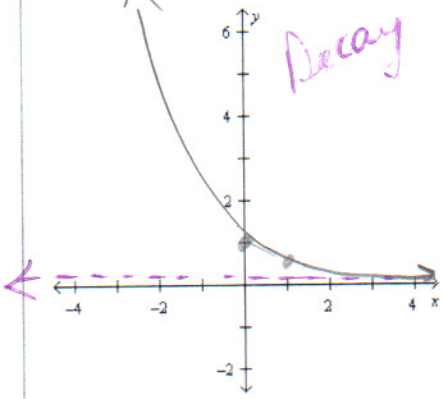
② Key point: y-intercept

③ [illegible] $x=1$

Domain $(-\infty, \infty)$

Range $(0, \infty)$

$f(x) = \left(\frac{1}{2}\right)^x$



Decay

$x=0 \quad y=1$

$x=1 \quad y=\frac{1}{2}$

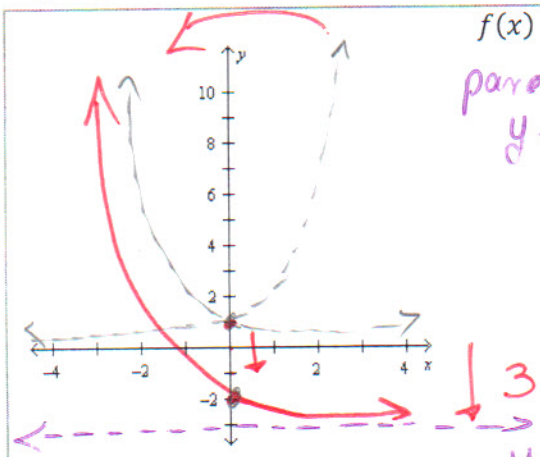
$y=0$

D: $(-\infty, \infty)$

R: $(0, \infty)$

Graphing Exponential Functions Using Transformations

$f(x) = 2^{-x} - 3$



parent: $y=2^x$

$y=-3$

D: $(-\infty, \infty)$

R: $(-3, \infty)$

$f(x-h)+k$ ↕ ↔

List transformations:

① "-x" reflect across y-axis

② "-3" shift down 3 ↓3

(asymptote also moves ↓3)

③ • Show Key pt
• Asymptote
• Curve

The Natural Exponential Number

A chap named Leonard Euler named this irrational number $e = 2.71828$ .... Applications include the naturally occurring processes of continuous growth and decay and may also be used to model any growth/decay that is continuous.

The ***number e*** is defined as the number the number in the expression:

$$\left(1+\frac{1}{n}\right)^{n}$$

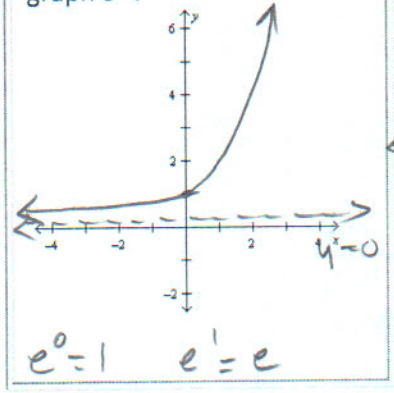
The main thing we need to recognize that and exponential equation is expressed with a base value of e"

$$y = e^{x}$$

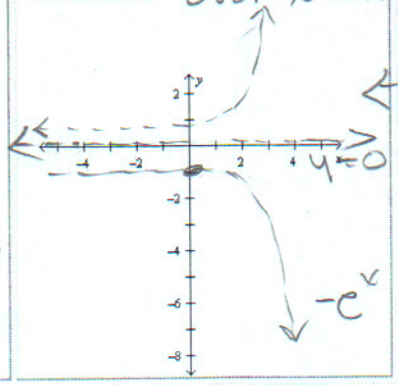
Using transformations, sketch the graph of the function:

$$f(x) = -e^{x-3}$$

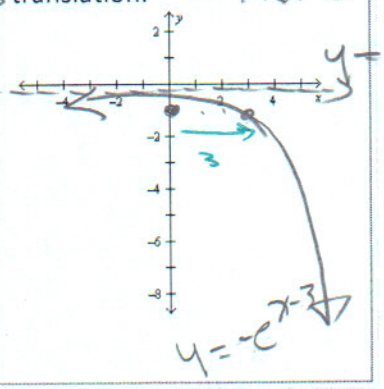
graph e^x:



reflection: over x-axis



translation: → R+3



Solve an Exponential Equation: ? find x ?? ★ Bases equal than exponents are equal

$$4^{2x-1} = 8^{x}$$

Can we write bases to be =?

$4 = 2^2$ $\quad 8 = 2^3$

So....

$2^{2(2x-1)} = 2^{3(x)}$

Bases equal - exponents equal: Alg I.

$2(2x-1) = 3x$

$4x - 2 = 3x$

$\boxed{x = 2}$

$$e^{-x^2} = (e^{x})^{2}\frac{1}{e^{3}}$$

Use rules of exponents & clean up

$e^{-x^2} = e^{2x}(e^{-3})$ $\quad a^m a^n = a^{m+n}$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x - 3$

$x^2 + 2x - 3 = 0$ Quadratic!!

$(x+3)(x-1) = 0$

$\boxed{x = -3, 1}$