Precalculus
Lesson 8.3: Law of Cosines
Mrs. Snow, Instructor

Obey the Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Last section we looked at the law of sines. There are two other situations where the law of sines will not work; here we will use the **Law of Cosines:**

Case 3 – Two sides and the angle included between the two sides are known (SAS).
Case 4 – Three sides are known (SSS).

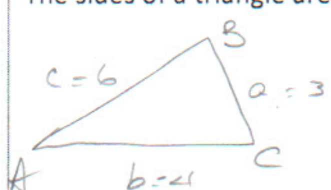
A word of caution: Once you find that 1st angle or side you can use Law of Sines to solve for the remaining pieces. NOTE: $\sin\theta$ = acute ∡ not necessarily the Obtuse ∡ that is the actual answer!!

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

EXAMPLE: SSS

The sides of a triangle are: $a = 3, b = 4, and\ c = 6$. Find the angles of the triangle



$3^2 = 4^2 + 6^2 - 2(4)(6)\cos A$

$\cos A \sim .8958\ldots$

$\boxed{A = 26.38°}$

$4^2 = 3^2 + 6^2 - 2(3)(6)\cos B$

$\cos B \sim .805\ldots$

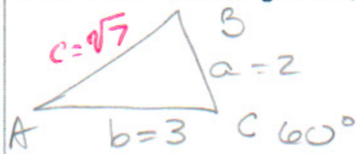
$\boxed{B = 36.34°}$

$6^2 = 3^2 + 4^2 - 2(3)(4)\cos C$

$\cos C \sim 117.28$

Check = 180??

SAS: Solve the triangle ABC, where $\angle C = 60°, a = 2, and\ b = 3$



$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 60$

$c^2 = 7 \rightarrow \boxed{c = \sqrt{7}}$

$2^2 = 3^2 + 7 - 2(3)(\sqrt{7})\cos A$

$\cos A \sim .75 \ldots$

$\boxed{A = 40.9°}$

$3^2 = 2^2 + 7 - 2(2)(\sqrt{7})\cos B$

$\cos B \sim .188 \ldots$

$\boxed{B = 79.1°}$

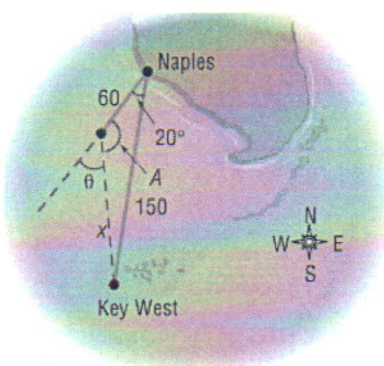
Navigation

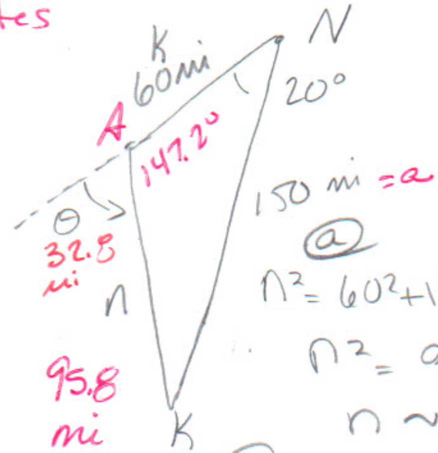
A motorized sail boat leaves Naples, Florida bound for Key West, 150 miles away. Maintaining a constant speed of 15 mph, but encountering heavy crosswinds and strong currents, the crew finds after 4 hours that the sailboat is off course by 20°.

a) How far is the sailboat from Key West at this time? 95.8 mi

b) Through what angle should the sailboat turn to correct its course 32.8°

c) How much time has been added to the trip because of this? Assume a constant speed of 15 mph. ~ 24 minutes





$\frac{15\,mi}{hr}(4\,hr) = 60$

ⓐ

$n^2 = 60^2 + 150^2 - 2(60)(150)(\cos 20)$

$n^2 = 9185.53$

$n \sim 95.8\,mi$

ⓒ Extra miles

$150 - 60 - 95.8 = 5.8\,mi$

$5.8\,mi\left(\frac{1\,hr}{15\,mi}\right) = .39\,hr\left(\frac{60\,min}{hr}\right)$

$= 24\,min$ Approx

ⓑ

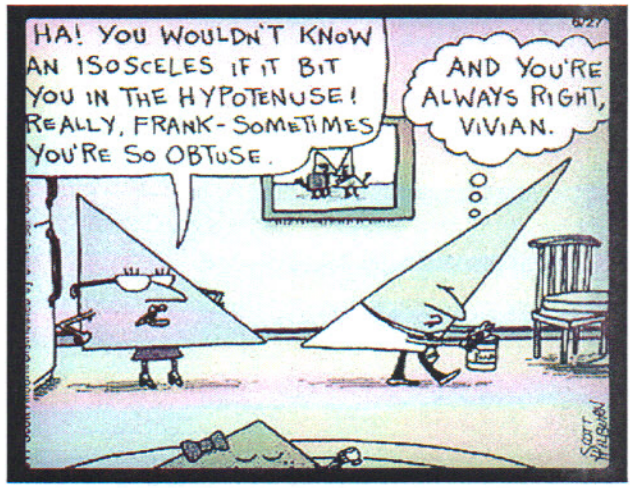
$150^2 = 60^2 + 95.8 - 2(60)(95.8)\cos A$

$\cos A \sim -.840 \ldots$

$A = 147.2°$ $\theta = 180 - 147.2$

$= 32.8°$

Precalculus
Lesson 8.4: Area of a Triangle
Mrs. Snow, Instructor



If we know two sides of a triangle and the included triangle we may apply the general formula for the area of a triangle (SAS).

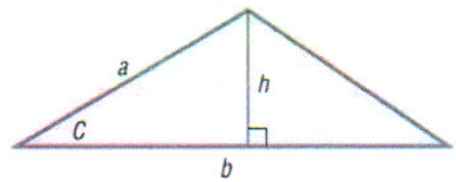
$$\sin C = \frac{h}{a}$$

solving for h:

$$h = \text{a} \sin C$$

so area is:

$$K = \frac{1}{2}bh = \frac{1}{2}ab \sin C$$



To find area of a triangle knowing SAS

$$K = \frac{1}{2}ab \sin C$$
$$K = \frac{1}{2}ac \sin B$$
$$K = \frac{1}{2}\text{bc} \sin A$$

From the law of cosines comes **Heron's Formula** that may be used to find the area of a triangle if only given the lengths of the three sides (SSS):

For a triangle with sides of lengths $a, b, and\ c$, it will have a **semiperimeter** of:

$$\boldsymbol{s = \frac{1}{2}(a + b + c)}$$

the area of the triangle is:

$$\boldsymbol{K = \sqrt{s(s-a)(s-b)(s-c)}}$$

Find the area of a triangle whose sides are
$a = 4, b = 5, c = 7$

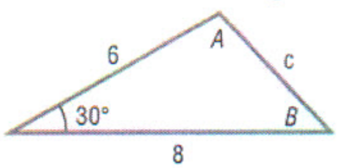
Three Sides

Use semiperimeter & Heron's Formula

$$S = \tfrac{1}{2}(a+b+c)$$
$$= \tfrac{1}{2}(4+5+7) = 8$$

$$K = \sqrt{8(8-4)(8-5)(8-7)} = \sqrt{96} = \boxed{4\sqrt{6}\ \text{units}^2}$$

Find the area of the triangle:



Use general formula as we have 2 sides & angle.

$$\text{Area} = \tfrac{1}{2}(6)(8)\sin 30^\circ$$
$$= \boxed{12\ \text{units}^2}$$