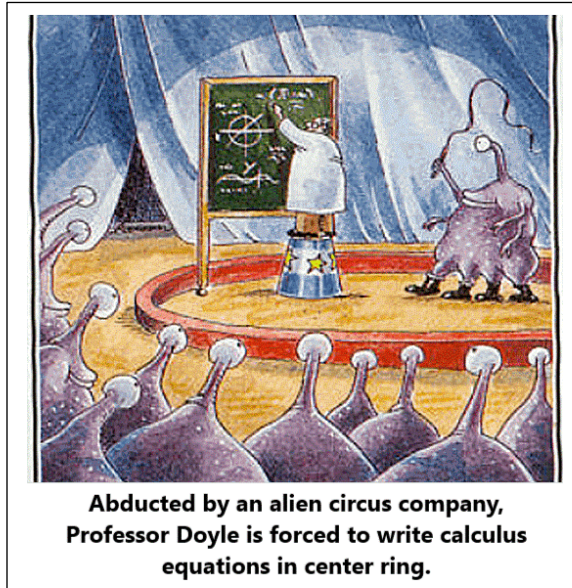


Calculus
Lesson 4.6: Numerical integration
Mrs. Snow, Instructor



Some functions simply do not have antiderivatives are elementary functions. In these cases, the Fundamental Theorem of Calculus cannot be applied and we need to use an approximation technique.

One technique, Trapezoidal Rule, can help us to approximate a definite integral by using breaking the area of the region defined by the definite integral into n trapezoids.

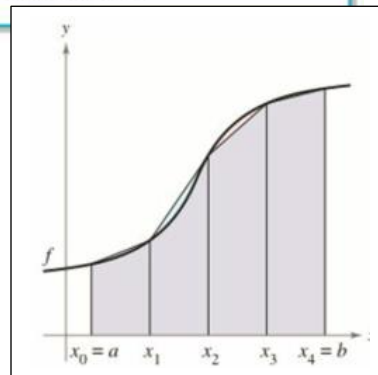
THEOREM 4.17 THE TRAPEZOIDAL RULE

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x) dx$ is given by

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \rightarrow \infty$, the right-hand side approaches $\int_a^b f(x) dx$.

Partition the interval $[a, b]$ into n subintervals,
each of width $\Delta x = \frac{b-a}{n}$



Approximation with the Trapezoid Rule

Use the Trapezoid Rule to approximate the following. Use $n=4$ and $n=8$.

$$\int_0^{\pi} \sin x \, dx$$

Simpson's Rule: For Simpson's Rule we approximate the function using parabolic approximations. The parabolas intersect with points on the subintervals of the given function. To develop Simpson's Rule for approximating a definite integral, you again partition the interval $[a, b]$ into n subintervals, each of width $\Delta x = (b - a)/n$, however this time we need to have **n be even.**

THEOREM 4.19 SIMPSON'S RULE

Let f be continuous on $[a, b]$ and let n be an even integer. Simpson's Rule for approximating $\int_a^b f(x) \, dx$ is

$$\int_a^b f(x) \, dx \approx \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \rightarrow \infty$, the right-hand side approaches $\int_a^b f(x) \, dx$.

Approximation with Simpson's Rule

Use Simpson's Rule to approximate the following. Use $n=4$ and $n=8$.

$$\int_0^{\pi} \sin x \, dx$$