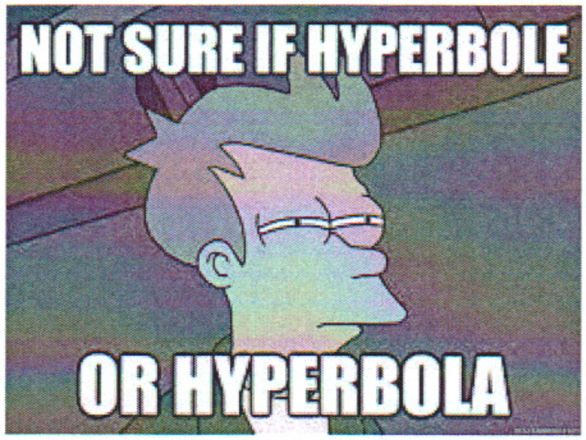
Precalculus

Lesson 10.4: The Hyperbola

Mrs. Snow, Instructor



A **hyperbola** is the collection (locus) of all points In the plane, the difference of whose distances from two fixed points, called the foci, is a constant.

Equation of a Hyperbola Centered about the origin with Transverse Axis along the x-axis

Who is first? Who is positive? } x

run x   y rise

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

a = run   b = rise

Transverse Axis variable is first & positive.

where

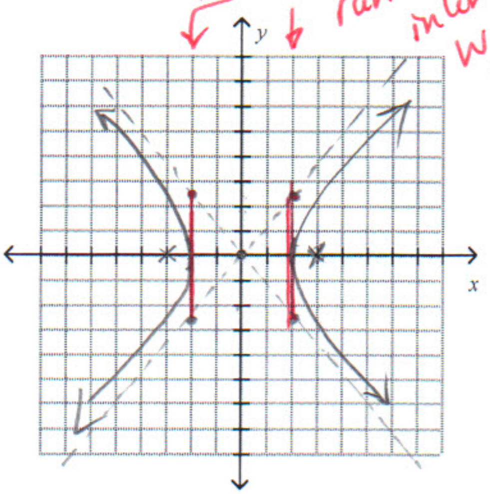
$$b^2 = c^2 - a^2$$

center at $(0, 0)$; foci at $(\pm c, 0)$; and vertices at $(\pm a, 0)$

two oblique asymptotes: $y = \pm\dfrac{b}{a}x$

Find an equation of the hyperbola with center at the origin, one focus at (3, 0) and one vertex at (−2, 0). Graph

rise/run is in line with vertex



Transverse axis Horizontal

$a = 2 \quad c = 3$

$a^2 = 4 \quad c^2 = 9$

$b^2 = c^2 - a^2$

$b^2 = 9 - 4 = 5 \rightarrow b = \pm\sqrt{5}$

Asymptote slope: $\frac{\text{rise}}{\text{run}}$

$\pm\frac{b}{a} = \pm\frac{\sqrt{5}}{2}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\boxed{\frac{x^2}{4} - \frac{y^2}{5} = 1}$$

Analyze the equation; find the center, transverse axis, vertices, and foci. Graph.

$\frac{x^2}{16} - \frac{y^2}{4} = 1$ x - first/positive (Horizontal)

a^2 b^2

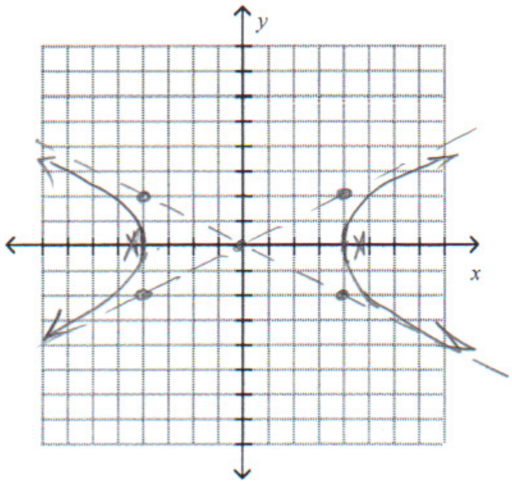
$a^2 = 16$ $b^2 = 4$ $b^2 = c^2 - a^2$

$a = 4$ $b = 2$ $4 = c^2 - 16$

Asymptote $= \pm \frac{2}{4}$ $20 = c^2$

$\pm 2\sqrt{5} = c$

(between 4 & 5)



Note! Slope $= \frac{\text{rise}}{\text{run}}$ is in line w/ vertex

Center: $(0, 0)$ w/Horizontal Transverse Axis

Vertices: $(0, \pm 4)$

foci: $(0 \pm 2\sqrt{5})$

Transv. axis Horizontal

Equation of a Hyperbola; Center at (0, 0); Transverse Axis along the y-axis

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$b^2 = c^2 - a^2$$

center at $(0, 0)$; foci at $(0, \pm c)$; and vertices at $(0, \pm a)$

$$\textit{two oblique asymptotes}: \; y = \pm \frac{a}{b} x$$

Analyze the equation, find the center, transverse axis, vertices, and foci and graph:

$$\frac{y^2}{4} - \frac{4x^2}{4} = \frac{4}{4} \qquad \frac{y^2}{4} - \frac{x^2}{1} = 1$$

y is first

Vertical transverse axis;

center $(0,0)$ $\quad a^2 = 4 \quad a = 2$

Vertices $(0, \pm 2)$ $\quad b^2 = 1 \quad b = 1$

foci $(0, \pm \sqrt{5})$

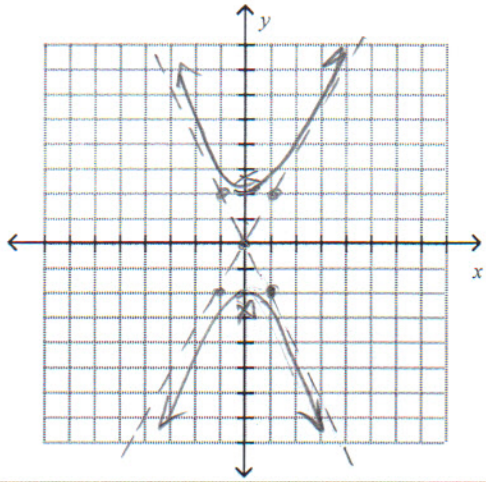
$b^2 = c^2 - a^2$

$1 = c^2 - 4$

$5 = c^2; \ \underline{c = \pm\sqrt{5}}$

Asymptote slope

$\pm \frac{\text{rise}}{\text{run}} = \frac{2}{1}$



Find an equation of the hyperbola having one vertex at $(0,2)$ and foci at $(0, -3)$ and $(0, 3)$. Graph.

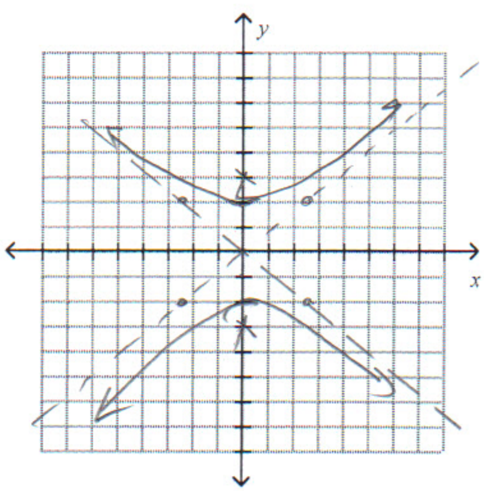
$c = 3 \qquad a = 2$

$b^2 = 9 - 4$

$b^2 = 5$

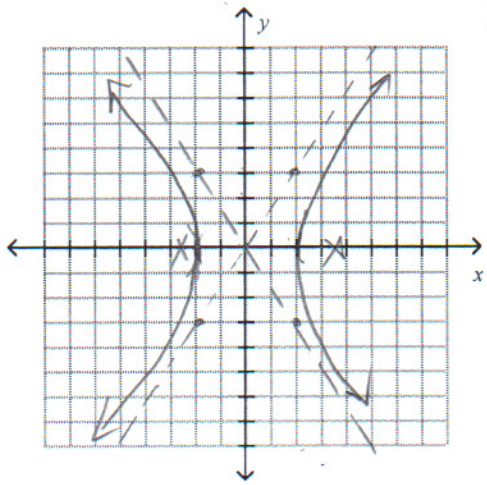
vertical $\rightarrow \dfrac{y^2}{4} + \dfrac{x^2}{5} = 1$

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{\sqrt{5}}$



Analyze the equation, find the center, transverse axis , vertices, foci, and asymptotes and graph:

$$\frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36} \qquad \frac{x^2}{4} - \frac{y^2}{9} = 1$$



Horizontal

$a^2 = 4$ $b^2 = 9$

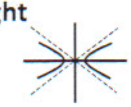
$a = 2$ $b = 3$ $9 = c^2 - 4$

$13 = c^2$

Slope $= \pm\frac{3}{2}$

$\pm\sqrt{13} = c$

$3 < \sqrt{13} < 4$

Hyperbolas at a center of (h, k) Transverse Axis Parallel to a Coordinate Axis $b^2 = c^2 - a^2$		
Opens	**Opens left and right** **Transverse axis** **x-axis** 	**Opens up and down** **Transverse axis** **y-axis** 
Form:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center:	(h, k)	(h, k)
Vertices	$(h+a, k) and (h-a, k)$	$(h, k+a) and (h, k-a)$
Slope of Asymptotes	$\pm\frac{b}{a}$	$\pm\frac{a}{b}$
Equation of Asymptotes	$\boldsymbol{y - k = \pm\frac{b}{a}(x-h)}$*	$\boldsymbol{y - k = \pm\frac{a}{b}(x-h)}$*
Foci	$(h+c, k), (h-c, k))$	$(h, k+c), (h, k-c)$

**The homework will ask for the equation of the asymptote. For the quiz and test, <u>all you will be expected to answer is the slope of the asymptote line.</u>*

Find an equation for the hyperbola with center at $(1, -2)$, one focus at $(4, -2)$, and one vertex at $(3, -2)$. Graph the equation by hand.

$(-2,-2)$

$(-1,-2)$

Horizontal Transverse Axis

Distances

$c = 4 - 1 = 3$

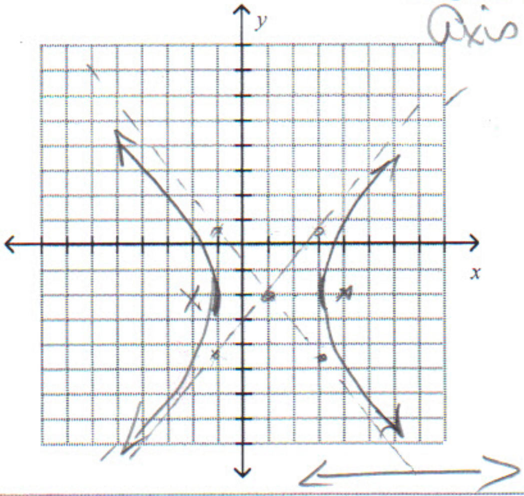
$a = 3 - 1 = 2$

$a^2 = 4$

Slope $= \frac{\sqrt{5}}{2}$

$b^2 = c^2 - a^2$

$b^2 = 9 - 4 = 5$



$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{5} = 1$$

Analyze the equation, find the center, transverse axis , vertices, foci, and asymptotes and graph:

$-x^2 + 4y^2 - 2x - 16y + 11 = 0$

$+16 - 1$

$4y^2 - 16y + 16 - x^2 - 2x - 1 = -11$

$4(y^2 - 4y + 4) - (x^2 + 2x + 1) = 4$

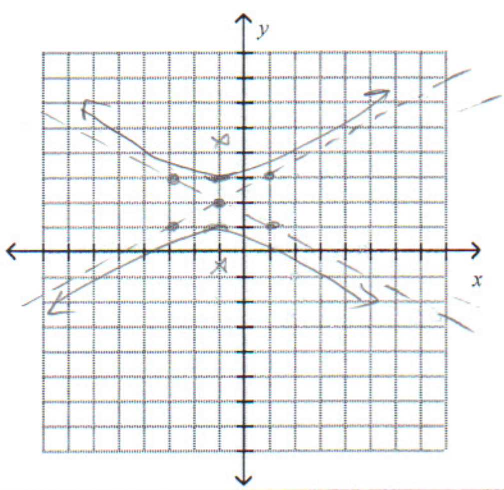
$$\frac{\cancel{4}(y-2)^2}{\cancel{4}\ 1} - \frac{(x+1)^2}{4} = \frac{\cancel{4}\ 1}{\cancel{4}}$$

y x

$$\frac{(y-2)^2}{1} - \frac{(x+1)^2}{4} = 1$$

+ $(h,k) = (-1, 2)$

+ y positive vertical axis



$a^2 = 1$ $b^2 = 4$

$a = 1$ $b = 2$

$4 = c^2 - 1$

$5 = c^2 \Rightarrow c = \pm\sqrt{5}$

+ vertices $(-1, 3)(-1, 2)$

+ foci $(-1, 2 \pm \sqrt{5})$

$M = \frac{1}{2}$