Optimization Problems

1. If a ball is thrown vertically upward from the top of an 80-foot tower with initial speed 72 ft/s, its height h above the ground t seconds later is $80+72t-16t^2$ feet. Find the greatest height reached by the ball.

2. Determine two positive numbers whose sum is 12, and such that the product of one of them and the square of the other is a maximum.

3. A rancher wishes to fence in a rectangular plot of land and divide it into two corrals with a fence parallel to two of the sides. If she has 300 yd of fencing available, how large an area can the plot have?

4. An open box is to be formed from a square sheet of metal 3 feet on a side by cutting equal squares out of the corners and bending up the flaps. Find the dimensions of the box if its volume is to be as large as possible.

5. Find the greatest volume of a cylinder inscribed in a cone of base radius 6 and height 9. (Hint: Use similar triangles.) (See figure below on left.)

6. Find the greatest volume of a cylinder that can be inscribed in a sphere with a radius a. (See figure below on right.)