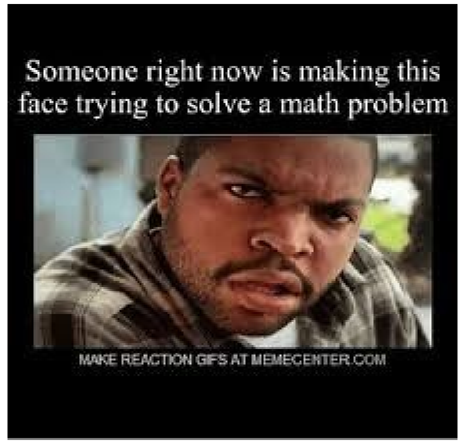
Calculus
Lesson: 3.3 Increasing and Decreasing Functions and the First Derivative Test
Mrs. Snow, Instructor

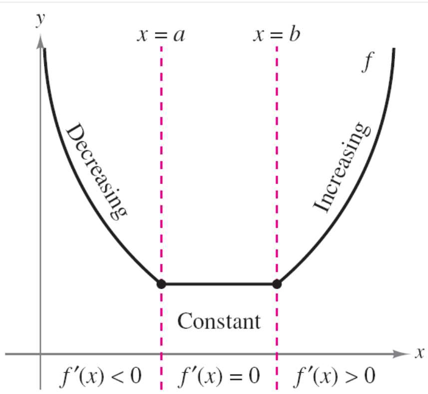


In this section we will study how derivatives can be used to classify relative extrema as either relative minima or relative maxima.

DEFINITIONS OF INCREASING AND DECREASING FUNCTIONS

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



The derivative is related to the slope of a function.

A function is increasing if, *as x moves to the right*, its graph moves up, and is decreasing if its graph moves down.

THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

WARNING!! If our function f is to be continuous, always remember to verify the domain of f before determining the critical points. If could be that where f' is a "DNE" critical point, f has a domain restriction!!!

Intervals on Which *f* is Increasing or Decreasing

Find the open intervals on which f(x) is increasing or decreasing.

$$f(x) = x^3 - \frac{3}{2}x^2$$

GUIDELINES FOR FINDING INTERVALS ON WHICH A FUNCTION IS INCREASING OR DECREASING

Let f be continuous on the interval (a, b). To find the open intervals on which f is increasing or decreasing, use the following steps.

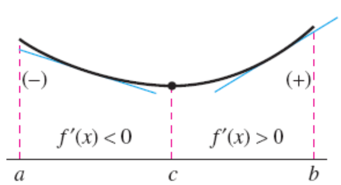
1. Locate the critical numbers of f in (a, b), and use these numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Use Theorem 3.5 to determine whether f is increasing or decreasing on each interval.

These guidelines are also valid if the interval (a, b) is replaced by an interval of the form $(-\infty, b)$, (a, ∞), or $(-\infty, \infty)$.

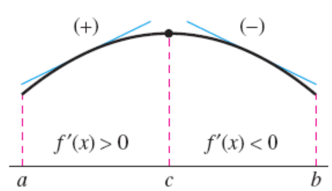
THEOREM 3.6 THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

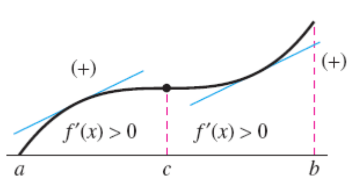
1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.

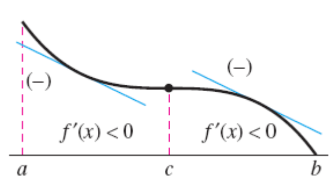


Relative minimum



Relative maximum





Neither relative minimum nor relative maximum

Applying the First Derivative Test

Find the relative extrema of the function f(x) in the interval (0,2π). ***(check domain of $f(x)$)***

$$f(x) = \frac{1}{2}x - \sin x$$

Find the relative extrema of

$$f(x) = \left(x^2 - 4\right)^{2/3}$$

Find the relative extrema of

$$f(x) = \frac{x^4 + 1}{x^2}$$