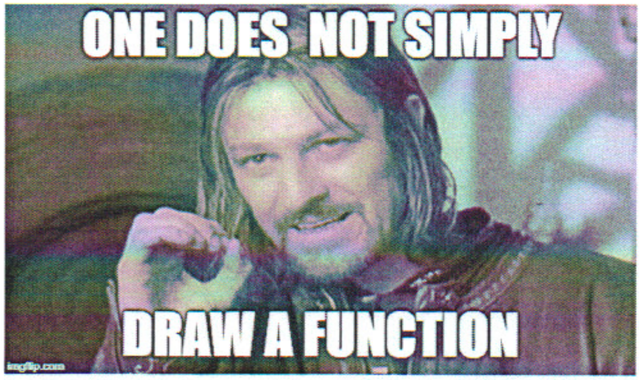
Precalculus

Lesson: 2.1 What is a Function and Lesson 2.2: Graphs of Functions

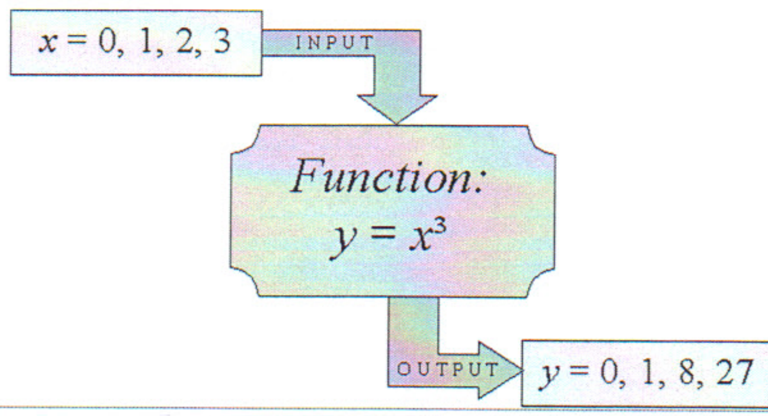
Mrs. Snow, Instructor

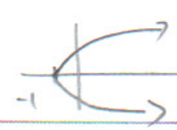


Lesson 2.1

"Working Definition" of Function"

A **function** is a relation for which each value from the set of the first components (independent variable) of the ordered pairs is associated with **exactly one value** from the set of second components (dependent variable) of the ordered pair. When we think of function equations, for every input x there exactly one output value of y. There are no x repeaters.



Determine whether the equation is a function.	
$y = \frac{1}{2}x - 3$	$x = y^2 - 1$
$y = mx + b$ / slope intercept form.	Solve for y
	$y^2 = x + 1$
	$y = \pm\sqrt{x+1}$
	$\sqrt{x}$ ← IR+
function	 Not function

For the given function evaluate: $f(x) = 2x^2 - 3x$ for:

x so: when x is equal to "__" plug in for all x.

(a) $f(3)$ (b) $f(x) + f(3)$ (c) $3f(x)$ (d) $f(-x)$

(e) $-f(x)$ (f) $f(3x)$ (g) $f(x+3)$ (h) $\dfrac{f(x+h) - f(x)}{h}$ $h \neq 0$

a) $f(3) = 2(3^2) - 3(3)$
$= 2(9) - 9$
$= 18 - 9 = 9 = f(3)$

b) $f(x) + f(3) =$
$= 2x^2 - 3x + 9$

c) $3f(x) = 3(2x^2 - 3x)$
$= 6x^2 - 9x$

d) $f(-x) = 2(-x)^2 - 3(-x)$
$= 2x^2 + 3x$

e) $-f(x) = -(2x^2 - 3x)$
$= -2x^2 + 3x$

f) $f(3x) = 2(3x)^2 - 3(3x)$
$= 2(9x^2) - 9x = 18x^2 - 9x$

g) $f(x+3) =$
$2(x+3)^2 - 3(x+3) =$
$2(x^2 + 6x + 9) - 3x - 9 =$
$2x^2 + \cancel{12x} + 18 - \cancel{3x} - 9 =$
$2x^2 + 9x + 9$

h) $\dfrac{f(x+h) - f(x)}{h} =$

$= \dfrac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$

$= \dfrac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- 3x} - 3h \cancel{- 2x^2} \cancel{+ 3x}}{h}$

$= \dfrac{2h^2 + 4xh - 3h}{h} =$

$= \dfrac{\cancel{h}(2h + 4x - 3)}{\cancel{h}} = 2h + 4x - 3$

Domain of a Function

Three points to remember!!

1. Denominator cannot equal zero
2. Anything under a square root has to be greater than or equal to zero, what if the square root is located in a denominator?
3. If no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number.

Find the domain: *Remember interval notation only!!!*

$f(x) = \dfrac{x+4}{x^2-2x-3}$	$g(x) = x^2 - 9$	$h(x) = \sqrt{3-2x}$
$x^2-2x-3 \neq 0$ $(x-3)(x+1) \neq 0$ $x-3 \neq 0 \quad x+1 \neq 0$ $x \neq 3 \quad x \neq -1$ Ans $(-\infty,-1) \cup (-1,3) \cup (3,\infty)$ $-1 \quad 3$	All Real $(-\infty,\infty)$ (parabola)	≥ 0 $3-2x \geq 0$ $\dfrac{-2x}{-2} \geq \dfrac{-3}{-2}$ $x \leq 3/2$ $3/2$ $(-\infty, 3/2]$

If we have two functions, we can use different techniques to combine them into one function

If f and g are functions:
The **sum** $f + g$ is the function defined by

$$\boxed{(f+g)(x) = f(x) + g(x)}$$

Domain: $f \cap g$

The **difference** $f - g$ is the function defined by

$$\boxed{(f-g)(x) = f(x) - g(x)}$$

Domain: $f \cap g$

The **product** $f \cdot g$ is the function defined by

$$\boxed{(f \cdot g)(x) = f(x) \cdot g(x)}$$

Domain: $f \cap g$

The **quotient** $\dfrac{f}{g}$ is the function defined by

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0}$$

Domain: $\{x \mid g(x) \neq 0\}$, $\cap$ domain of f $\cap$ domain of g

Combinations of Functions and Their Domains:

Let $f(x) = 2x^2 + 3 \quad and\ g(x) = 4x^3 + 1$

1. Find the functions $(f+g)(x),\ (f-g)(x),\ (f\cdot g)(x), and\ \left(\frac{f}{g}\right)(x)$ and determine their domains.

$(f+g)(x) = 2x^2+3+4x^3+1$

$= \underline{\underline{4x^3+2x^2+4}}$ D: $(-\infty,\infty)$

$(f-g)(x) = (2x^2+3)-(4x^3+1)$

$= 2x^3+3-4x^3-1$

$= \underline{\underline{-4x^3+2x^3+2}}$

D: $(-\infty,\infty)$

$(f\cdot g)(x) = (2x^2+3)(4x^3+1)$

$= 8x^5+2x^2+12x^3+3$

D: $(-\infty,\infty)$

$\frac{f}{g}(x) = \frac{2x^2+3}{4x^3+1}$

& $4x^3+1 \neq 0$

$4x^3 \neq -1$

$x^3 \neq -1/4$

$x \neq \sqrt[3]{-1/4} \approx -.63 \rightarrow$ D: $(-\infty,-.63)\cup(-.63,\infty)$

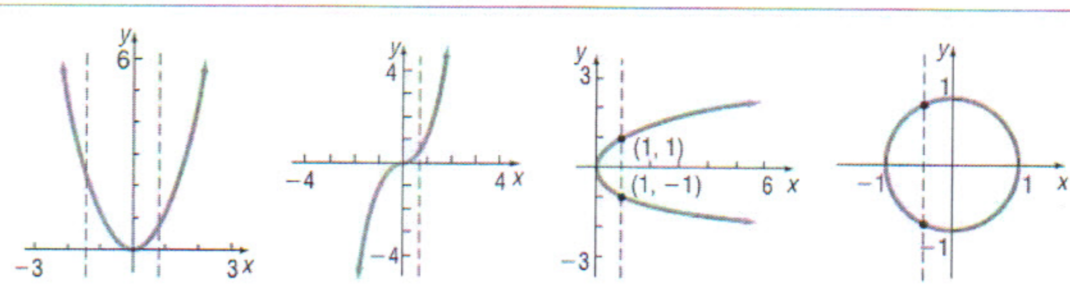
Lesson 2.2 - Graphs of Functions

Sometimes a visual representation, a graph, of a relationship is easier to understand.

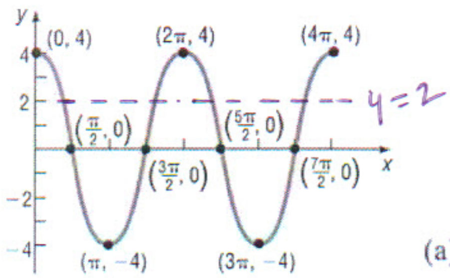
The Vertical Line Test is a technique to verify if a graph represents a function.

Vertical Line Test: The graph of a function cannot contain two points with the same x-coordinate and different y-coordinates.

Identify the graphs that represent a function and the domains for all:



Obtaining Information from the Graph of a Function



(a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?
$= 4$ $= 0$ $= -4$

(b) What is the domain of f? $[0, 4\pi]$

(c) What is the range of f? $[-4, 4]$

(d) List the intercepts. (Recall that these are the points, if any, where the graph crosses or touches the coordinate axes.) $(0,4)$, $(\frac{\pi}{2}, 0)$ $(\frac{3\pi}{2}, 0)$, $(\frac{5\pi}{2}, 0)$ & $(7\pi/2, 0)$

(e) How many times does the line $y = 2$ intersect the graph? — 4 times

(f) For what values of x does $f(x) = -4$? $x = \pi, 3\pi$

(g) For what values of x is $\underbrace{f(x) > 0}_{\text{Above x-axis}}$? $[0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \frac{5\pi}{2}) \cup (\frac{7\pi}{2}, 4\pi]$

Obtaining Information about the Graph of a Function

Consider the function: $f(x) = \dfrac{x+1}{x+2}$

(a) Find the domain of f. (Denominator $\neq 0$) $x+2 \neq 0$, $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

(b) Is the point $\left(1, \frac{1}{2}\right)$ on the graph of f? $\frac{1}{2} \stackrel{?}{=} \frac{1+1}{1+2} = \frac{2}{3}$ NO

(c) If $x = 2$, what is $f(x)$? What point is on the graph of f?

$f(2) = \dfrac{2+1}{2+2} = \dfrac{3}{4}$

Average Cost Function

The average cost $\overline{C}$ of manufacturing x computers per day is given by the function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Determine the average cost of manufacturing:

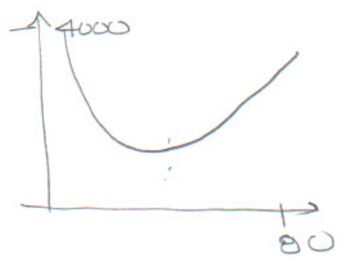
(a) 30 computers in a day
(b) 40 computers in a day
(c) 50 computers in a day
(d) Graph the function $\overline{C} = \overline{C}(x)$, $0 < x \leq 80$.
(e) Create a TABLE with TblStart = 1 and ΔTbl = 1. Which value of x minimizes the average cost?

a) $\overline{C}(30) = .56(30^2) - 34.39(30) + 1212.57 + \frac{20000}{30}$
$= \boxed{\$1351.34}$

b) $\overline{C}(40) = .56(40^2) - 34.39(30) + 1212.57 + \frac{20000}{40}$
$= \boxed{\$1232.97}$

c) $\overline{C}(50) = .56(50^2) - 34.39(50) + 1212.57 + \frac{20000}{50}$
$= \boxed{\$1293.07}$

d)



e) minimum
$x = 41$ computers
$\overline{C} = 1231.74$