Precalculus
Lesson: 4.3 Complex Zeros; Fundamental Theorem of Algebra
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Not all quadratic equations have real solutions.

A variable in the complex number system is referred to as a **complex variable.** A **complex polynomial function** f of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are complex numbers, $a_n \neq 0$, n is a nonnegative integer, and x is a complex variable. As before, a_n is called the **leading coefficient** of f. A complex number r is called a **complex zero** of f if $f(r) = 0$.

If we look at the complex number system, every quadratic equation has at least one complex solution; remember rational and irrational roots are in fact complex numbers. We just don't write them in the complex form. The fact that each polynomial function will have a complex solution brings about an important theorem.

Fundamental Theorem of Algebra.

Every complex polynomial function $f(x)$ of degree $n \geq 1$ has at least one complex zero.

Another important theorem states that if we have the solution $a + bi$ then we must also have the solution $a - bi$.

Conjugate Pairs Theorem.

Let $f(x)$ be a polynomial function whose coefficients are real numbers. If $r = a + bi$ is a zero of f, the complex conjugate $\bar{r} = a - bi$ is also a zero of f.

and a corollary

A polynomial function f of odd degree with real coefficients has at least one real zero.

Using the Conjugate Pairs Theorem

A polynomial function f of degree 5 whose coefficients are real numbers has the zeros 1, $5i$, and $1 + i$. Find the remaining two zeros.

Example:

Find a polynomial function of degree 4 whose coefficients are real numbers and that has the zeros of $1, 1, -4 + i$

Given that $x = \pm 2$ are solutions, find the remaining complex zeros of the polynomial function.

$$f(x) = x^4 + 2x^3 + x^2 - 8x - 20$$