Calculus
Lesson 2.2: Basic Differentiation Rules
and Rate of Change
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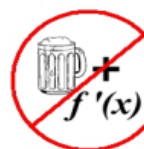
In 2.1 we reviewed the limit definition to find derivatives. Now we are ready move on to several "differentiation rules" that allow you to find the derivatives without the *direct* use of the limit definition!
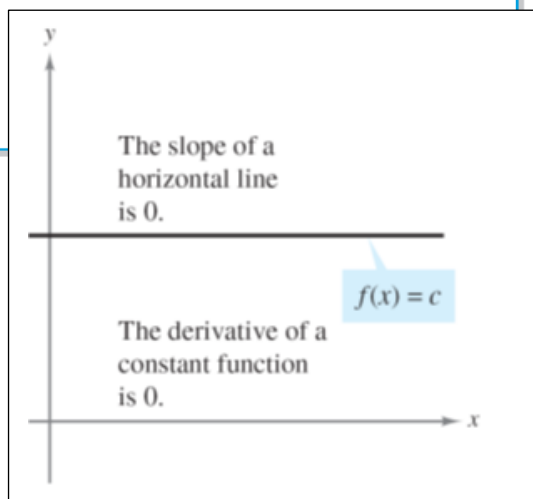
THEOREM 2.2 THE CONSTANT RULE

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0.$$

(See Figure 2.14.)

y

The slope of a horizontal line is 0.

$f(x) = c$

The derivative of a constant function is 0.

x

Using the Constant Rule:

Function	Derivative
$y = 7$	$\frac{dy}{dx} =$
$f(x) = 0$	$f'(x) =$
$s(t) = -3$	$s'(t) =$
$y = k\pi^2$ (k is constant)	$y' =$

THEOREM 2.3 THE POWER RULE

If n is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = nx^{n-1}.$$

For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

Using the power rule

Function	Derivative

a. $f(x) = x^3$

b. $g(x) = \sqrt[3]{x}$

c. $y = \dfrac{1}{x^2}$

Finding the Slope of the Graph:

Find the slope of the graph of $f(x) = x^4$ when:

a. $x = -1$ 　　　　　　　　b. $x = 0$ 　　　　　　　　c. $x = 1$

Finding an Equation of a Tangent Line

Find and equation of the tangent line to the graph of $f(x) = x^2$ when $x = 2$

Using the Constant Multiple Rule

$$y = \frac{2}{x}$$

$$f(t) = \frac{4t^2}{5}$$

$$y = 2\sqrt{x}$$

$$y = \frac{1}{2\sqrt[3]{x^2}}$$

$$y = -\frac{3x}{2}$$

Using Parentheses When Differentiating

Be careful when solving a problem:

$$y = \frac{7}{3x^{-2}}$$

vs.

$$y = \frac{7}{(3x)^{-2}}$$

$$y = \frac{5}{2x^3}$$

vs.

$$y = \frac{5}{(2x)^3}$$

Using the Sum and Difference Rules

Function	Derivative

a. $f(x) = x^3 - 4x + 5$

b. $g(x) = -\dfrac{x^4}{2} + 3x^3 - 2x$

Derivatives of Sine and Cosine Functions

We also have formulas for our trig functions, two of which are below:

$y = 2\sin x$

$$y = \frac{\sin x}{2} = \frac{1}{2}\sin x$$

$y = x + \cos x$

Rates of Change

$$Average\ velocity = \frac{Change\ in\ Distance}{Change\ in\ Time} = \frac{\Delta y}{\Delta x}$$

Make sure to distinguish between instantaneous and average velocity. All else fails read the directions!

If a Billiard ball is dropped from a height of 100 ft, its height s at time t is given by $s = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity over each of the following time intervals.

$a)\ [1, 2]$ $b)\ [1, 1.5]$ $c)\ [1, 1.1]$

Remember, slope is rate of change. If the rate of change is distance per time, then the slope is a velocity. Since the derivative is the rate of change at a point P, we have an instantaneous rate of change or an instantaneous velocity:

Instantaneous Velocity

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = s'(t)$$

where the position function is given by:

$$s(t) = \frac{1}{2} g t^2 + v_o t + s_o$$

S_o is the initial height of the object, v_o is the initial velocity of the object, and g is the acceleration due to gravity. On Earth, the value of g is approximately
$$-32 \, feet \, per \, second \text{ or } -9.8 \, meters \, per \, second.$$

Using the derivative to Find Velocity

- At time t=0, a diver jumps from a platform diving board that is 32 feet above the water.
- The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t is measured in seconds.
  a. When does the diver hit the water?
  b. What is the diver's velocity at impact?