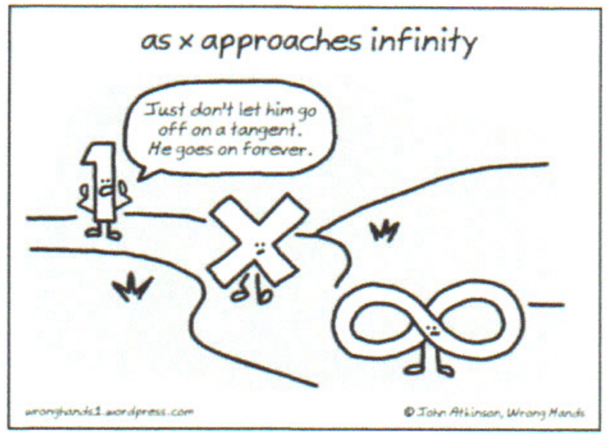
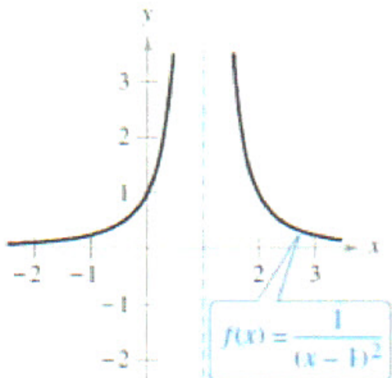
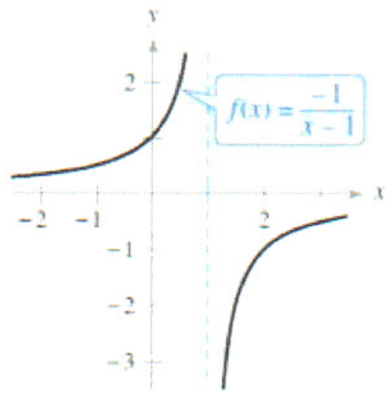
Calculus
Lesson 1.5: Infinite Limits
Mrs. Snow, Instructor



Determining Limits from a Graph:





If it were possible to extend the graphs toward positive and negative infinity, you would see that each graph becomes arbitrarily close to the vertical line $x = 1$. This line is a **vertical asymptote** of the graph of *f*.

The vertical asymptotes occur at $x = 1$, why???

Definition of Vertical Asymptote

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

THEOREM 1.14 Vertical Asymptotes

Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c such that $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by

$$h(x) = \frac{f(x)}{g(x)}$$

has a vertical asymptote at $x = c$.

Determine all vertical asymptotes of the graph of each function: Asymptotes occur where denominator = 0.

a. $f(x) = \dfrac{1}{2(x+1)}$ → $2(x+1) = 0$

$x + 1 = 0$

$\boxed{x = -1}$

b. $f(x) = \dfrac{x^2+1}{x^2-1}$ → $(x+1)(x-1) = 0$

$\boxed{x = -1,\ x = 1}$

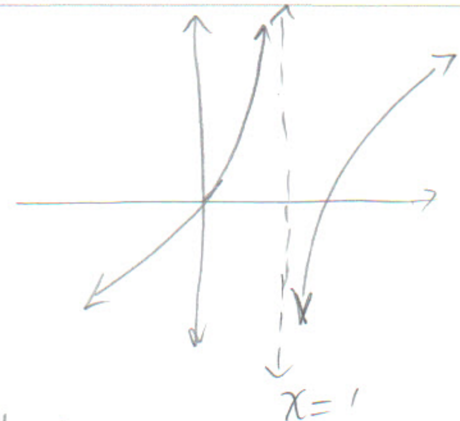
c. $f(x) = \cot x = \dfrac{\cos x}{\sin x}$ → $\sin x = 0$

at integer multiples of π → $\boxed{x = n\pi}$

Find each limit:

$\lim\limits_{x \to 1^-} \dfrac{x^2 - 3x}{x-1}$ and $\lim\limits_{x \to 1^+} \dfrac{x^2 - 3x}{x-1}$

$\boxed{\lim\limits_{x \to 1^-} \infty \qquad \lim\limits_{x \to 1^+} = -\infty}$



at $x = 1$ asymptote,

cannot do direct substitution

So; ......

THEOREM 1.15 Properties of Infinite Limits

Let c and L be real numbers, and let f and g be functions such that

$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L.$$

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x \to c} [f(x)g(x)] = -\infty, \quad L < 0$
3. Quotient: $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$ [see Example 5(d)].

Because of the above properties the following examples are true: Why???? (Break down the limits using above properties.)

Determining limits:

a.

$$\lim_{x \to 0} \left(1 + \frac{1}{x^2}\right) = \infty.$$

$$\lim_{x \to 0} 1 + \lim_{x \to 0} \frac{1}{x^2} =$$

$$1 + \infty =$$

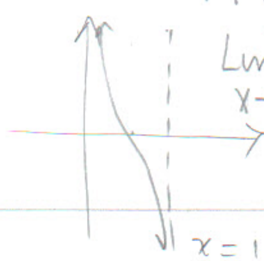
$$\boxed{\infty}$$

b.

$$\lim_{x \to 1} \frac{x^2 + 1}{\cot \pi x} = 0.$$

$$\lim_{x \to 1^-} x^2 + 1 =$$

$$1 + 1 = 2$$



$$\lim_{x \to 1^-} \cot \pi x = -\infty$$

$$\text{so: } \frac{2}{-\infty} = \boxed{0}$$

c.

$$\lim_{x \to 0^+} 3 \cot x = \infty$$

$$\left(\lim_{x \to 0^+} 3\right)\left(\lim_{x \to 0^+} \cot x\right) =$$

$$(3)(\infty) =$$

$$\infty$$

So is the answer $\pm\infty$? Or is it Does Not Exist?????? That is the frustration of moving into a higher level class, varying textbooks, online homework programs, etc. First you are taught that the only numbers out there are the counting numbers. Then somewhere along the way you are told that there are more numbers out there. First fractions, then irrational numbers. That is not the end of it, in high school you're introduced to imaginary numbers, and you are taught that as we head off to infinity an actual numerical value does not exist, so DNE is the correct answer, now……. Yes, we call it as it is $\pm\infty$. **WHY?** I guess this is what is called is "baby steps" in the learning process.

When we describe the limit as $\pm\infty$ it describes the end behavior of the function. End behavior is in important concept in understanding what a function looks like at very large or very small values.