

Precalculus
Lesson 000: Stuff You Should Already Know: Algebra II Review
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Polynomials

A **polynomial** in one variable is an algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad (1)$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are constants,* called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is a variable. If $a_n \neq 0$, it is the **leading coefficient**, and n is the **degree** of the polynomial.

$$-8x^3 + 4x^2 + 6x + 2$$

- a) What is the degree of the polynomial?
- b) What is the leading coefficient?

Special products

Difference of Two Squares

$$(x - a)(x + a) = x^2 - a^2 \quad (2)$$

Squares of Binomials, or Perfect Squares

$$(x + a)^2 = x^2 + 2ax + a^2 \quad (3a)$$

$$(x - a)^2 = x^2 - 2ax + a^2 \quad (3b)$$

Cubes of Binomials, or Perfect Cubes

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 \quad (4a)$$

$$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3 \quad (4b)$$

Difference of Two Cubes

$$(x - a)(x^2 + ax + a^2) = x^3 - a^3 \quad (5)$$

Sum of Two Cubes

$$(x + a)(x^2 - ax + a^2) = x^3 + a^3 \quad (6)$$

Multiply the factors:

$$(x - 3)(x + 3)$$

$$(x + 2)^2$$

$$(2x + 1)(3x + 4)$$

$$(x - 2)(x^2 + 2x + 4)$$

Factor:

$$x^4 - 16$$

$$x^3 - 1$$

$$9x^2 - 6x + 1$$

$$x^2 + 4x - 12$$

$$3x^2 + 10x - 8$$

$$x^3 - 4x^2 + 2x - 8$$

Simplifying Rational Expressions:

$$\frac{x^2 + 4x + 4}{x^2 + 3x + 2}$$

$$\frac{x^3 - 8}{x^3 - 2x^2}$$

$$\frac{8 - 2x}{x^2 - x - 12}$$

Multiplying and Dividing Rational Expressions

$$\frac{x^2 - 2x + 1}{x^3 + x}$$

$$\frac{\frac{x + 3}{x^2 - 4}}{\frac{x^2 - x - 12}{x^3 - 8}}$$

Adding and Subtracting Rational Expressions

$$\frac{x^2}{x^2 - 4} - \frac{1}{x}$$

$$\frac{x}{x^2 + 3x + 2} + \frac{2x - 3}{x^2 - 1}$$

Simplify:

$$\frac{\frac{1}{2} + \frac{3}{x}}{\frac{x+3}{4}}$$

Quadratic Formula

Quadratic Formula

Consider the **quadratic** equation

$$ax^2 + bx + c = 0 \quad a \neq 0$$

If $b^2 - 4ac < 0$, this equation has no real solution.

If $b^2 - 4ac \geq 0$, the real solution(s) of this equation is (are) given by the **quadratic formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the solutions, if any, of the equation:

$$3x^2 - 5x + 1 = 0$$

Interval Notation

Let a and b represent two real numbers with $a < b$.

A **closed interval**, denoted by $[a, b]$, consists of all real numbers x for which $a \leq x \leq b$.

An **open interval**, denoted by (a, b) , consists of all real numbers x for which $a < x < b$.

The **half-open**, or **half-closed, intervals** are $(a, b]$, consisting of all real numbers x for which $a < x \leq b$, and $[a, b)$, consisting of all real numbers x for which $a \leq x < b$.

Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Write each inequality using interval notation

$$1 \leq x \leq 3$$

$$-4 < x < 0$$

$$x > 5$$

$$x \leq 1$$

