Precalculus Name______________________________ per____
Integral Practice

Indefinite Integrals

Review. The derivative with respect to x
If $f(x) = x^3 + 5$, then $f'(x) = 3x^2$

We can also write this using shorter notation as $\frac{d}{dx}x^3 + 5 = 3x^2$

The notation $\frac{d}{dx}$ means, "change per unit change in x" or "the derivative with respect to x."

It comes from the notation $\frac{\Delta y}{\Delta x}$ used to find slope. This same idea of a comparison of changes can be written as $\frac{dy}{dx}$.

Derivatives and integrals are inverse operations (like multiplication and division, they "undo" each other). To find an integral, also know as the "antiderivative," is to reverse the process of the derivative.

If $\frac{d}{dx}x^3 + 5 = 3x^2$, then $\int 3x^2 dx = x^3 + c$, where c is a constant.
Why do we need the c? Look at the following derivatives:

$$\frac{d}{dx}x^2 + 4 = 2x$$

$$\frac{d}{dx}x^2 + 8 = 2x$$

$$\frac{d}{dx}x^2 - 7 = 2x$$

If we reverse the process with an integral, we must use the c to show the infinite number of functions with the same derivative. $\int 2x dx = x^2 + c$

The collection of *all* antiderivatives of the function *f(x)* is called the **indefinite integral** of f with respect to x.

Formula

$$\int nx^k dx = \frac{nx^{k+1}}{k+1} + c$$

Example 1. Find the antiderivative $\int 5x^4 dx = ??$

Solution: $\int 5x^4 dx = \frac{5x^{4+1}}{4+1} + c = \frac{5x^5}{5} + c = x^5 + c$

Check: $\frac{d}{dx}x^5 + c = 5x^4$

Example 2. Find the antiderivative $\int y^3 dy$

Solution: $\int y^3 dy = \frac{y^{3+1}}{3+1} + c = \frac{y^4}{4} + c = \frac{1}{4}y^4 + c$

Check: $\frac{d}{dy}\frac{1}{4}y^4 + c = 4 \cdot \frac{1}{4}y^3 = y^3$

Example 3. Find the antiderivative $\int (3x^4 + 2x^3 + 4x + 1)dx$

Solution: $\int (3x^4 + 2x^3 + 4x + 1)dx = \frac{3x^5}{5} + \frac{2x^4}{4} + \frac{4x^2}{2} + \frac{1x}{1} + c = \frac{3}{5}x^5 + \frac{1}{2}x^4 + 2x^2 + x + c$

Check: $\frac{d}{dx}\frac{3}{5}x^5 + \frac{1}{2}x^4 + 2x^2 + x + c = 5 \cdot \frac{3}{5}x^4 + 4 \cdot \frac{1}{2}x^3 + 2 \cdot 2x^1 + 1x^0 = 3x^4 + 2x^3 + 4x + 1$

Section 1. Find the following antiderivatives. (5 points each)

1. $\int 4x^3 dx =$ ______________
2. $\int y^4 dy =$ ______________
3. $\int x^7 dx =$ ______________
4. $\int 2w^7 dw =$ ______________
5. $\int (2x + 7)dx =$ ______________
6. $\int (3x^2 - 4x + 5)dx =$ ______________
7. $\int (4x^2 + 2x - 8)dx =$ ______________
8. $\int (5x^2 - x)dx =$ ______________
9. $\int (4x^3 + 2x^2 - 8x)dx =$ ______________
10. $\int (x^6 + 2)dx =$ ______________
11. $\int -4x^3 dx =$ ______________
12. $\int (5x^3 + 3x)dx =$ ______________

Definite Integrals

Provided you can find an antiderivative of *f*, you now have a way to evaluate a definite integral without having to use the limit of a sum.

Theorem

If f is continuous and nonnegative on [a,b], then the area under the graph from a to b is $\int_a^b f(x)dx$

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

"The area bounded by $f(x) = 3x^2, y = 0, x = 1, x = 3$" can now be written as $\int_1^3 3x^2 dx$.

$$\int_1^3 3x^2 dx = x^3\Big|_1^3 = 3^3 - 1^3 = 27 - 1 = 26 \text{ units}^2$$

*It is not necessary to include the constant c when doing definite integrals because

$$\int_a^b f(x)dx = F(x)+c\Big|_a^b = (F(b)+c)-(F(a)+c) = F(b)+c-F(a)-c = F(b)-F(a)$$

Example 4. Find the area under the graph from a to b.

$$f(x) = x+1; a = 0, b = 3$$

$$\int_0^3 (x+1)dx = \tfrac{1}{2}x^2 + x\Big|_0^3 = [\tfrac{1}{2}(3)^2 + 3] - [\tfrac{1}{2}(0)^2 + 0] = \frac{9}{2} + 3 - 0 = \frac{9}{2} + \frac{6}{2} = \frac{15}{2} \text{ units}^2$$

Example 5. Find the area under the graph from a to b.

$$f(x) = 3 - x^2; a = -1, b = 3$$

$$\int_{-1}^3 (3-x^2)dx = 3x - \tfrac{1}{3}x^3\Big|_{-1}^3 = [3(3) - \tfrac{1}{3}(3)^3] - [3(-1) - \tfrac{1}{3}(-1)^3] = (9-9) - (-3+\tfrac{1}{3}) = 0 - \frac{-8}{3} = \frac{8}{3} \text{ units}^2$$

<u>Section 2</u> Find the area under the graph f from a to b. (10 points each)

13. $f(x) = 4 - x; a = -1, b = 2$ 13. ____________________

14. $f(x) = 5 - 3x^2; a = -2, b = 2$ 14. ____________________

15. $f(x) = x^2 - 4x; a = 0, b = 4$ 15. ____________________

16. $f(x) = 3x^3 - 4x^2; a = 1, b = 2$ 16. ____________________