Precalculus
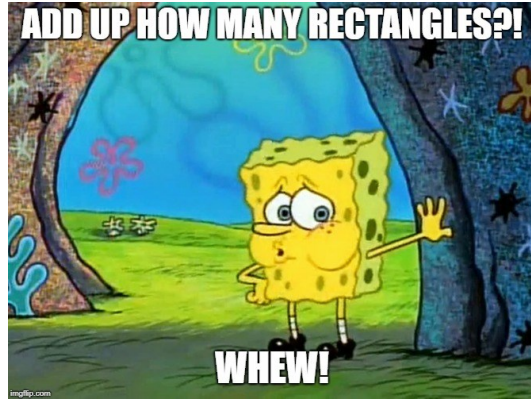**Lesson 14.5: The Area Problem: The Integral**
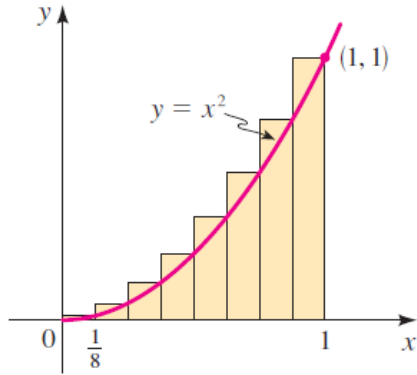**Mrs. Snow, Instructor**

In geometry we found area of polygons. We had set formulas such as the area of a rectangle is length times width. A triangular area is found by calculating ½ the length of the base times the height, and so on. Calculus is used to deal with area problems that have regions containing curved boundaries. Here we can go back to our simple formula for the area of a rectangle and use it to estimate the area of a region under a curve.

Estimating an Area Using Rectangles: Use rectangles to estimate the area under the parabola $y = x^2$, from 0 to 1.
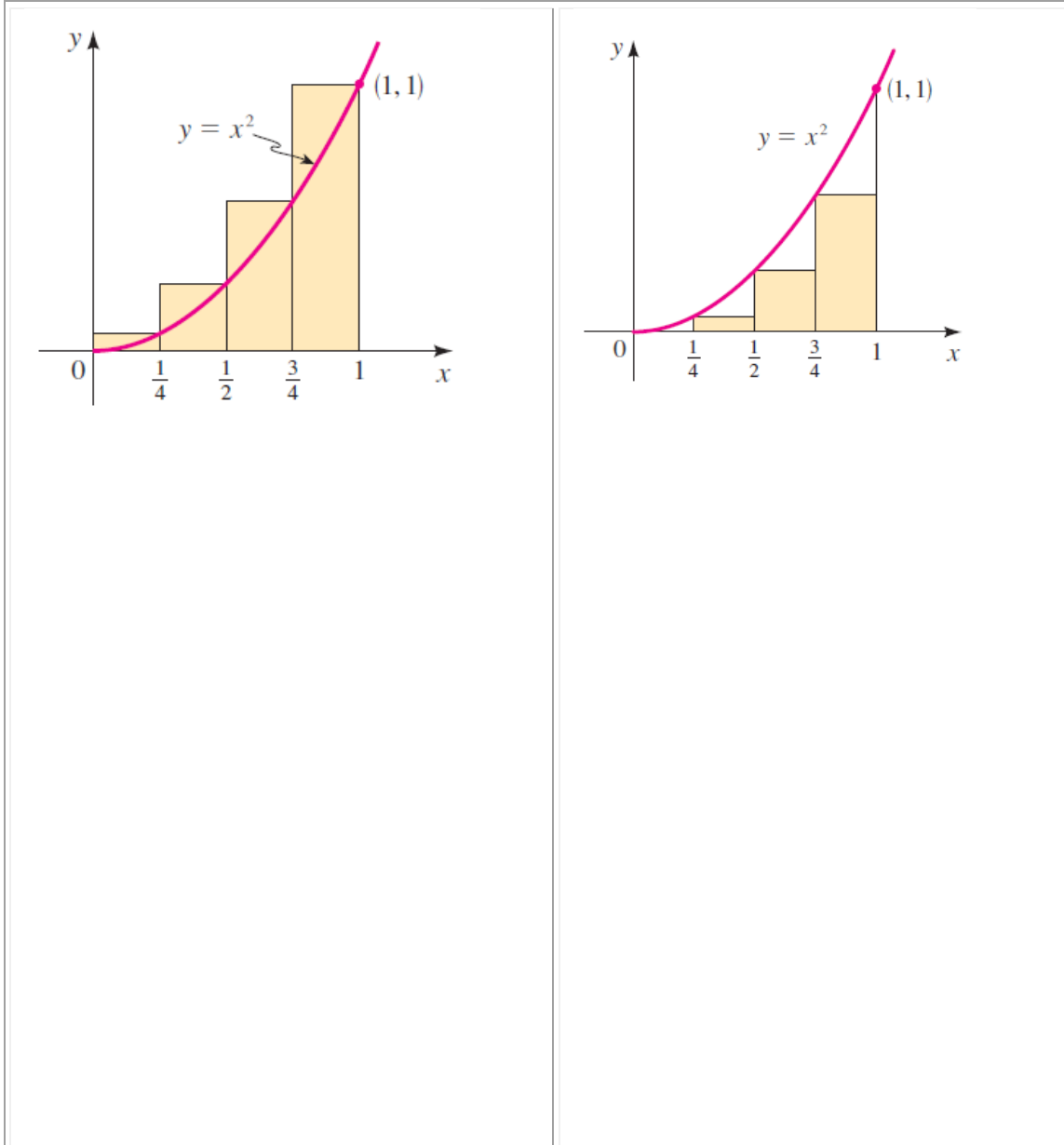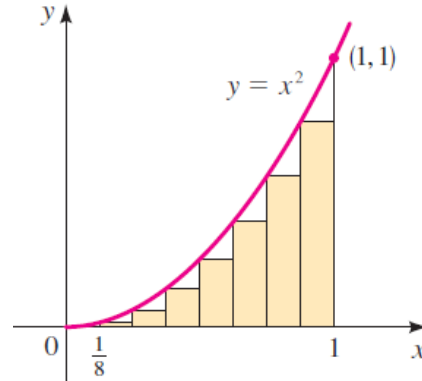
Same problem….smaller rectangles
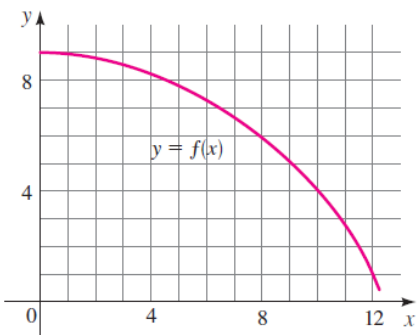
| Overestimate of area.	Underestimate of area.
(b) Using right endpoints	(a) Using left endpoints

The smaller the rectangular strips the more accurate the calculation of the area. This then opens up the door to take a limit as the number of rectangles goes to infinity.

	What if you were not given a definition of the function??????????? Making six rectangles from 0 to 12, how do we set up so to estimate the area?

Definition of Area

The area A of the region S that lies under the graph of a continuous function f is the limit of the sum of the areas of the approximating rectangles: *use right endpoints.*
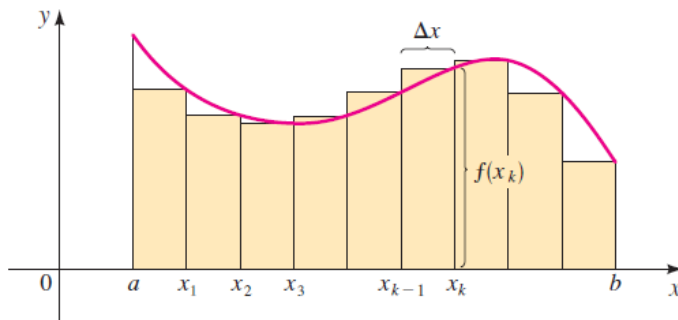
AREA=(height) x (width)

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f\left(x_k\right)\Delta x$$

Δx **is the width** of an approximating rectangle,
x_k **is the right endpoint** of the kth **rectangle**
$f(x_k)$ **is its height**.

$\boldsymbol{n}\, rectangles$
$region\, from\, x = a\ \ to\ \ x = b$

$\boldsymbol{width}$: $\Delta x = \frac{b-a}{n}$

$\boldsymbol{right\ endpoint}$: $x_k = a + k\Delta x$

$\boldsymbol{height}$: $f(x_k) = f(a + k\Delta x)$

These Summation Formulas are an extension of the summation concepts studied in chapter 12:

Summation Formulas!!

these will be used in solving of our area problems

$$\sum_{k=1}^{n} c = nc$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Finding the Area under a Curve:

Find the area of the region that lies under

$$y = x^2,\ 0 \le x \le 5$$

Find the area of the region that lies under

$$y = 4x - x^2 \quad 1 \le x \le 3$$