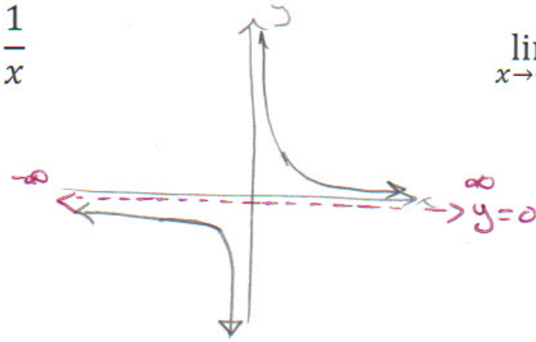
Precalculus
Lesson 14.4: Limits at Infinity
Mrs. Snow, Instructor

When you find there are limits at infinity

Just as the title of the lesson states, given a function $f(x)$ as x approaches infinity what happens to the value $f(x)$?

Find

$$\lim_{x\to\infty}\frac{1}{x} \qquad \lim_{x\to-\infty}\frac{1}{x}$$



As $x \to \infty$,
Limit $= 0$

As $x \to -\infty$
Limit $= 0$

The limit approaches horizontal asymptote: $y=0$

So we get a rule to remember:

If k is any positive integer, then:

Know!

$$\lim_{x\to\infty}\frac{1}{x^k}=0 \qquad \textit{and} \qquad \lim_{x\to-\infty}\frac{1}{x^k}=0$$

Strategy: to get variables into the $\frac{1}{x^k}$ form.

Evaluate:

$$= \lim_{x\to\infty}\frac{(3x^2 - x - 2)}{(5x^2 + 4x + 1)} \cdot \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$$

Multiply by "1" such that we get all x variables into the $\frac{1}{x^k}$ format

$$= \lim_{x\to\infty} \frac{\frac{3\cancel{x^2}}{\cancel{x^2}} - \frac{\cancel{x}}{x^{\cancel{2}}} - \frac{2}{x^2}}{\frac{5\cancel{x^2}}{\cancel{x^2}} + \frac{4\cancel{x}}{x^{\cancel{2}}} + \frac{1}{x^2}} =$$

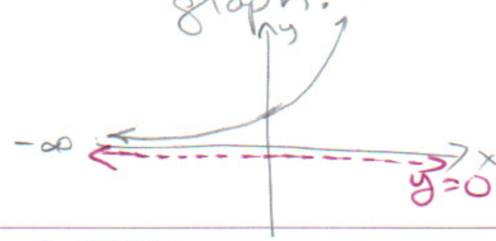
<u>How?</u> Use highest degree of x

$$= \lim_{x\to\infty} \frac{3 - \cancel{\frac{1}{x}}^{\to 0} - \cancel{\frac{2}{x^2}}^{\to 0}}{5 + \cancel{\frac{4}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}} = \boxed{\frac{3}{5}}$$

Finding a limit at negative infinity

$$\lim_{x\to-\infty} e^x = 0$$

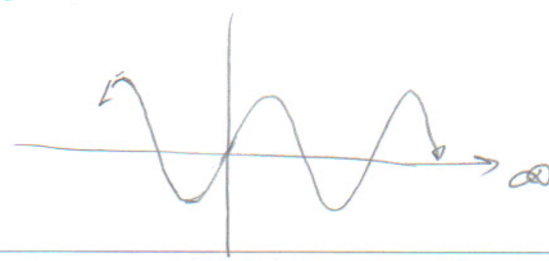
graph:



Can function at infinity have no limit??????

$$\lim_{x\to\infty} \sin x = \text{DNE}$$

graph:



Oscillates so never approaches a specific value.

In chapter 12 we studied sequences: $a_1, a_2, a_3, \ldots . a_n,$. Using limits we can determine the behavior of a sequence as n becomes large.

Convergent vs. divergent: Converge is when things come together from different directions so they eventually meet. Diverge is when things separate and go in different directions. Well, in sequences the term a_n may converge by approaching a number or it may not.....

Finding the Limit of a Sequence

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right) \cdot \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{1 + \frac{1}{n}_{\to 0}} = 1$$

Does the Sequence Converge or Diverge?

$a_n = (-1)^n$

Divergent

Expand:

$-1^1, -1^2, -1^3, -1^4, -1^5 \ldots$

$= -1, 1, -1, 1, -1 \ldots$

Divergent as it alternates between 1 & -1

Finding the Limit of a Sequence

Find the limit of the sequence given.

$$a_n = \frac{15}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

per formulas we can pull out constant

$$\lim_{n\to\infty} \frac{15}{n^3}\left[\frac{n(n+1)(2n+2)}{6}\right]$$

$$= \frac{15}{6} \lim_{n\to\infty}\left[\frac{n(n+1)(2n+2)}{n^3}\right]$$

Separate factors

$$= \frac{5}{2} \lim_{n\to\infty}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+2}{n}\right)$$

Separate: Numerator + Numerator over common denominator

$$= \frac{5}{2} \lim_{n\to\infty}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{\cancel{n}}{\cancel{n}} + \frac{1}{n}\right)\left(\frac{2\cancel{n}}{\cancel{n}} + \frac{2}{n}\right) =$$

$$\left(\frac{5}{\cancel{2}}\right)(\cancel{2}) = \boxed{5}$$