Precalculus
Lesson 14.2: Algebra Techniques for Finding Limits
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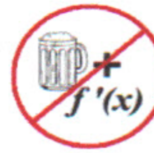
DON'T DRINK AND DERIVE



Mathematicians
Against
Drunk
Deriving

Know Your Limits!

Limit Laws

$\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$

$$1.\ \lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$$

$$2.\ \lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$$

$$3.\ \lim_{x\to a}\left[cf(x)\right]=c\lim_{x\to a} f(x)$$

$$4.\ \lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$$

$$5.\ \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$$

$$6.\ \lim_{x\to a}\left[f(x)\right]^n=\left[\lim_{x\to a} f(x)\right]^n$$

$$7.\ \lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$$

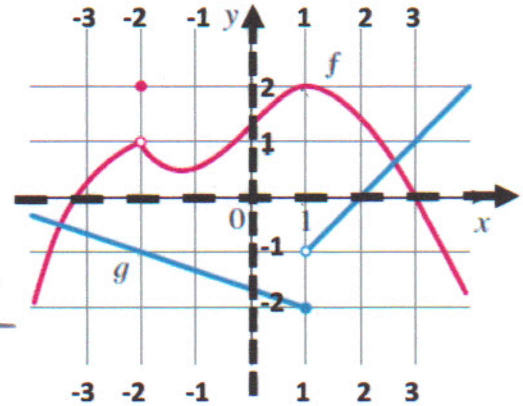
Using the Limit Laws

Use the limit laws and the graphs of f and g in Figure to evaluate the following limits, if they exist.

$\lim_{x\to -2}[f(x)+5g(x)] =$

$\lim_{x\to -2} f(x) + 5\lim_{x\to -2} g(x) =$

$1 + 5(-1) = \underline{-4}$



$\lim_{x\to 1}[f(x)g(x)] =$

$\lim_{x\to 1} f(x) \cdot \lim_{x\to 1} (g(x)) \leftarrow$ jump

$(2)(DNE) = \underline{DNE}$

$\lim_{x\to 2}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)} = \dfrac{1.4}{0} = \underline{DNE}$

$\lim_{x\to 1}[f(x)]^3 = \left(\lim_{x\to 1} f(x)\right)^3$

$= 2^3 = \underline{8}$

Some Special Limits

1. $\lim_{x\to a} c = c$
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a} x^n = a^n$
4. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

Limits by Direct Substitution:

$\lim_{x\to a} f(x) = f(a)$

May directly substitute "a" into equation and evaluate ☺

Using the Limit Laws: Evaluate the following limits.

$\lim_{x\to 5} 2x^2 - 3x + 4$

$\uparrow$ 5 $\uparrow$ 5

$= 2(5^2) - 3(5) + 4$

$2(25) - 15 + 4 = 39$

Finding Limits by Direct Substitution: Evaluate the following limits.

as long as you can directly substitute, do so!

$$\lim_{x\to -2} \frac{x^3+2x^2+1}{5-3x} = \frac{(-2)^3+2(-2)^2+1}{5-3(-2)} = \frac{-8+8+1}{11} = \frac{1}{11}$$

Note: We run into a problem if the expression is undefined!

$$\lim_{x\to 3} 2x^3-10x-8$$

$$2(3)^3-10(3)-8$$

$$54-30-8=16$$

$$\lim_{x\to -1} \frac{x^2+5x}{x^4+2}$$

$$=\frac{(-1)^2+5(-1)}{(-1)^4+2}$$

$$=\frac{1-5}{1+2}=\frac{-4}{3}$$

Finding a Limit by Canceling a Common Factor

$$\lim_{x\to 1}\frac{x-1}{x^2-1}$$

Cannot substitute => undefined

$$\Rightarrow \lim_{x\to 1}\frac{\cancel{(x-1)}\,1}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

Correct Format is required!

Must carry $\lim_{x\to a}$ through until direct substitution is possible

Finding a Limit by Simplifying

$$\lim_{h\to 0}\frac{(3+h)^2-9}{h} = \lim_{h\to 0}\frac{\cancel{9}+6h+h^2-\cancel{9}}{h} =$$

$$\lim_{h\to 0}\frac{\cancel{h}(6+h)}{\cancel{h}} = \lim_{h\to 0} 6+h = 6+0$$

$$=6$$

Finding a Limit by Rationalizing ⟷ multiply by the conjugate! $\times 1$

$$\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2} = \lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}\cdot\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$$

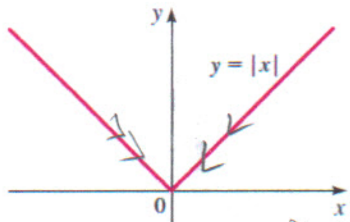
$$= \lim_{t\to 0}\frac{t^2+\cancel{9}+\cancel{3\sqrt{t^2+9}}-\cancel{3\sqrt{t^2+9}}-\cancel{9}}{(t^2)(\sqrt{t^2+9}+3)} =$$

$$\lim_{t\to 0}\frac{\cancel{(t^2)}\ 1}{\cancel{(t^2)}(\sqrt{t^2+9}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Comparing Right and Left Limits: Show that

Definition $|x| = \begin{cases} x & \text{for } x \geq 0 \quad \text{Right} \\ -x & x < 0 \quad \text{Left}\end{cases}$

$$\lim_{x\to 0}|x| = 0$$



$$\left.\begin{aligned}\lim_{x\to 0^+}|x| &= \lim_{x\to 0^+} x = 0 \\ \lim_{x\to 0^-}|x| &= \lim -x = 0\end{aligned}\right\}\text{ Same}$$

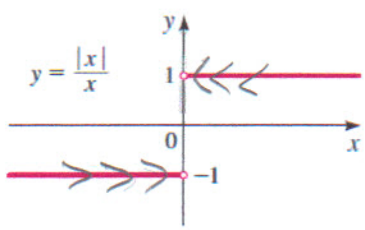
$\therefore \lim |x| = 0$

Definition example: for $x \geq 0$, let $x=5$: $|x| = x$, $|5| = 5$

for $x<0$, let $x=-5$: $|x| = -x$, $|-5| = -(-5) = 5$

Prove that

$$\lim_{x\to 0}\frac{|x|}{x} = \text{DNE}$$



Use definition:

$$\lim_{x\to 0^+}\frac{|x|}{x} = \lim_{x\to 0^+}\frac{x}{x} = 1$$

$$\lim_{x\to 0^-}\frac{|x|}{x} = \lim \frac{-x}{x} = \frac{-(-1)}{-1} = -1$$

Limits are different $\therefore$

DNE

The Limit of a Piecewise Defined Function

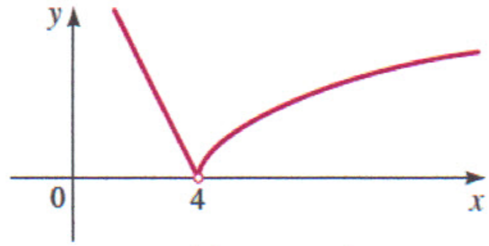
$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases}$$ from right / from left

find:
$\lim_{x\to 4} f(x)$ Algebraically!

$\lim_{x \to 4^+} \sqrt{x-4} = \sqrt{0} = 0$

$\lim_{x \to 4^-} 8-2(x) = 8-2(4) = 0$

$\therefore \lim_{x \to 4} f(x) = 0$



With graph we see $\lim_{x \to 4} f(x) = 0$