

Calculus
 Lesson 8.5: Using Partial Fraction
 Decomposition to Integrate
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In this lesson we will review a procedure for decomposing a rational function into simpler rational functions to which you can apply the basic integration formulas.

A review of partial fraction decomposition:

1. Distinct Linear Factors

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x - a_1} + \frac{A_2}{x - a_2} + \dots + \frac{A_n}{x - a_n}$$

2. Repeated Linear Factors

$$\frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_n}{(x - a)^n}$$

3. Distinct Quadratic Factors

$$\frac{Ax + B}{ax^2 + bx + c}$$

Finding a Partial Fraction Decomposition

$$\frac{x-13}{2x^2-7x+3} = \frac{A}{2x-1} + \frac{B}{x-3} = \left(\frac{5}{2x-1} + \frac{-2}{x-3} \right)$$

$$(2x-1)(x-3)$$

$$x-13 = A(x-3) + B(2x-1)$$

$$x=3 \quad -10 = B(6-1)$$

$$-10 = 5B \quad \underline{B = -2}$$

$$x=0$$

$$-13 = A(-3) - 2(-1)$$

$$-13 = -3A + 2$$

$$-15 = -3A$$

$$\underline{A = 5}$$

Finding an Integral with Partial Fractions

- Find

$$\int \frac{3x^4 + 1}{x^2 - 1} dx$$

Stop! Improper Fraction so divide!

$$\begin{array}{r} x^2 - 1 \overline{) 3x^4 + 1} \\ \underline{-3x^4 + 3x^2} \\ 3x^2 + 1 \\ \underline{-3x^2 + 3} \\ 4 \end{array}$$

$$\frac{4}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$4 = A(x-1) + B(x+1)$$

$$x=1 \quad 4 = 2B \quad B=2$$

$$x=-1 \quad 4 = -2A \quad -2 = A$$

$$= \int 3x^2 + 3 + \frac{4}{x^2 - 1} dx \quad \leftarrow \text{split denominator}$$

$$= \int 3x^2 + 3 + \frac{2}{x-1} - \frac{2}{x+1} dx$$

$$= \left(x^3 + 3x + 2 \ln|x-1| - 2 \ln|x+1| + C \right)$$

Finding Three Partial Fractions

- Find the general solution to

$$\frac{dy}{dx} = \frac{6x^2 - 8x - 4}{(x^2 - 4)(x-1)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$(x+2)(x-2)(x-1)$$

$$6x^2 - 8x - 4 = A(x+2)(x-1) + B(x-2)(x-1) + C(x-2)(x+2)$$

$$x=1 \quad 6-8-4 = C(-1)(3) \quad x=2$$

$$-6 = -3C$$

$$\underline{C=2}$$

$$x=-2$$

$$24-16-4 = A(4)(1) \quad 24+16-4 = B(-4)(-3)$$

$$4 = 4A$$

$$\underline{A=1}$$

$$36 = 12B$$

$$3 = B$$

$$\int dy = \int \frac{1}{x-2} + \frac{3}{x+2} + \frac{2}{x-1} dx$$

$$\left(y = \ln|x-2| + 3 \ln|x+2| + 2 \ln|x-1| + C \right)$$