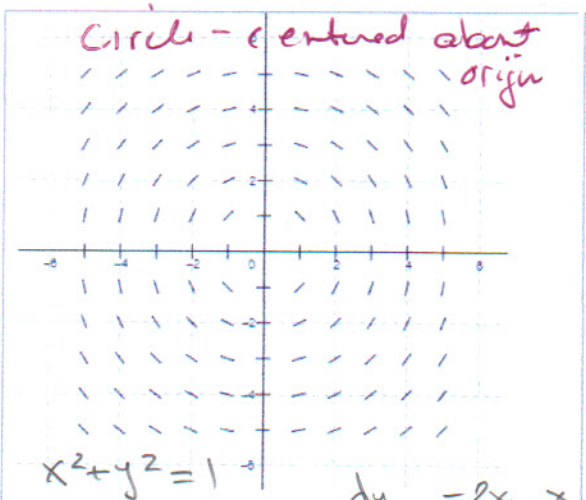
Calculus
Lesson 6.1: Slope Fields
Mrs. Snow, Instructor

Solving a differential equation analytically can be difficult or even impossible. Consider a differential equation if the form $y' = F(x, y)$ where $F(x, y)$ is some expression in terms of x *and* y. At each point (x, y), the differential equation determines the slope. Remember $y' = slope$. We can use a graphical method to learn about the solution of a differential equation. By drawing short line segments with the slopes as calculated by several points you will get what is called a **slope field or a direction field** for the differential equation.

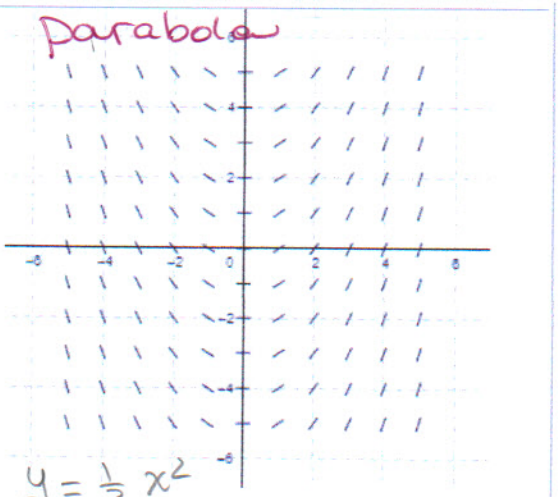
Tyler started to wish he had paid more attention in Algebra.

What does it look like?


Circle - centered about origin

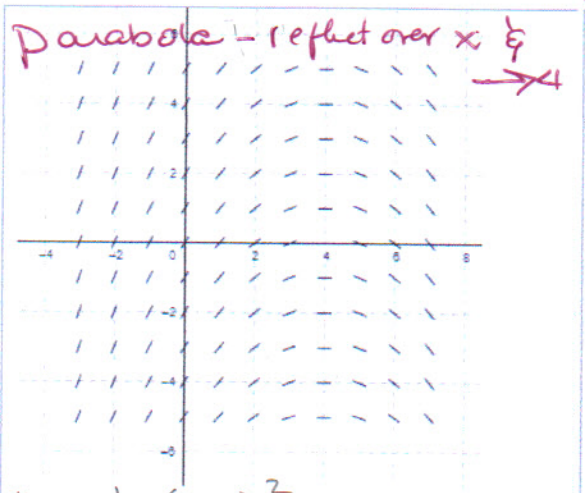
$x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$


Parabola

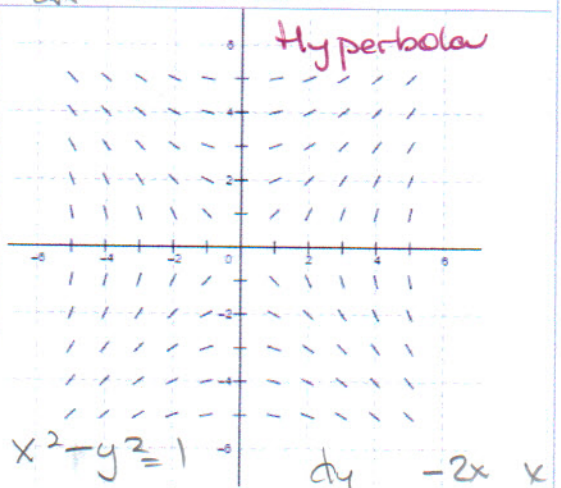
$y = \frac{1}{3}x^2$

$\frac{dy}{dx} = \frac{2}{3}x = m$


Parabola - reflect over x & →

$y = -\frac{1}{5}(x-4)^2$

$y' = -\frac{2}{5}(x-4)$


Hyperbola

$x^2 - y^2 = 1$

$2x - 2y\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

Sketch the slope fields

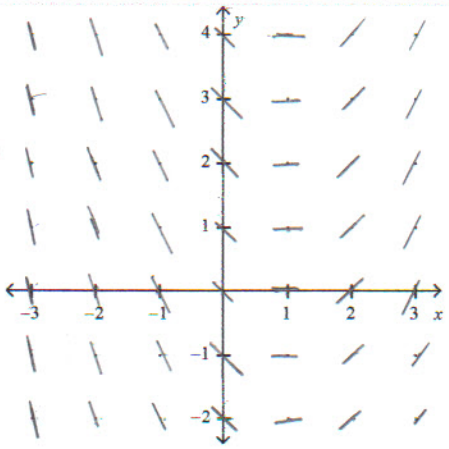
$$\frac{dy}{dx} = x - 1$$

x	m
-3	-4
-2	-3
-1	-2
0	-1
1	0
2	1
3	2

For x = -3 All m = -4

etc

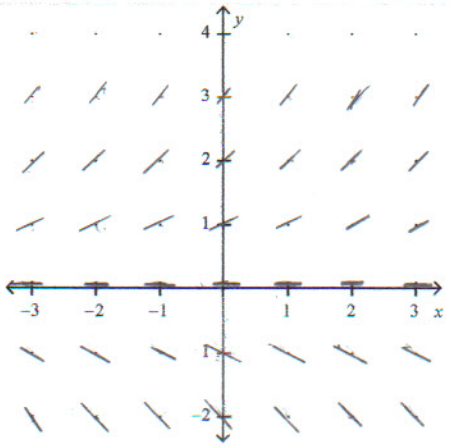
↓



$$\frac{dy}{dx} = \frac{1}{2}y$$

y	m
-3	$-\frac{3}{2}$
-2	-1
-1	$-\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$

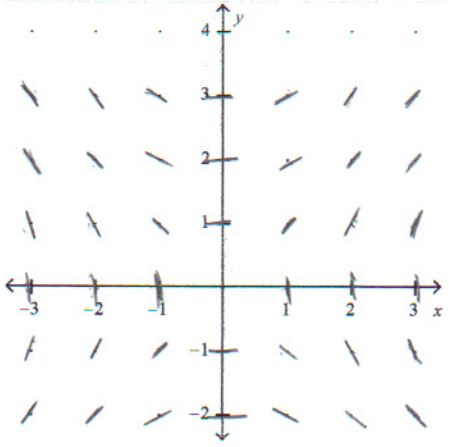
for a given y parallel slopes for all x



$$\frac{dy}{dx} = \frac{x}{y}$$

x	y	m
-1	-1	1
-2	-2	1
1	1	1
-1	1	-1
-2	2	-1
↓	↓	

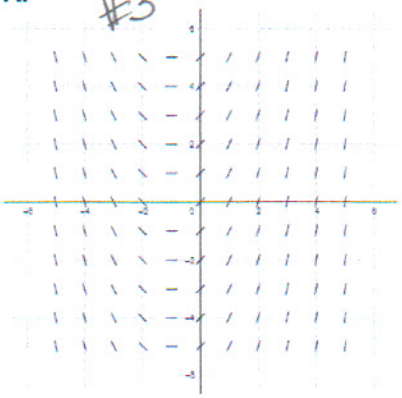
tedious!



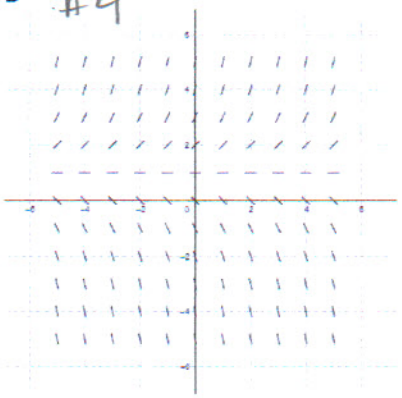
Identify the slope field

1. $\frac{dy}{dx} = 2y$ D — y=0, slope=0
2. $\frac{dy}{dx} = 2x$ C — x=0, m=0
3. $\frac{dy}{dx} = x + 1$ A — x=-1, m=0
4. $\frac{dy}{dx} = y - 1$ B — y=1, m=0
5. $\frac{dy}{dx} = x + y$ E — x=-y, m=0
6. $\frac{dy}{dx} = -\frac{x}{y}$ F — y=0, m=UNDEF, x=0, m=0

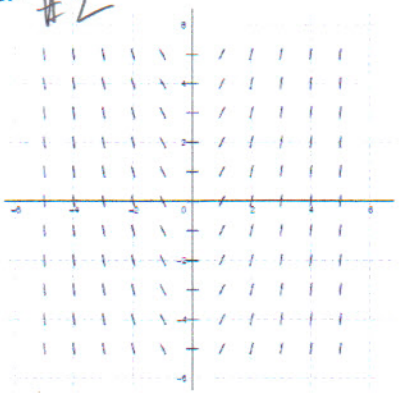
A. #3



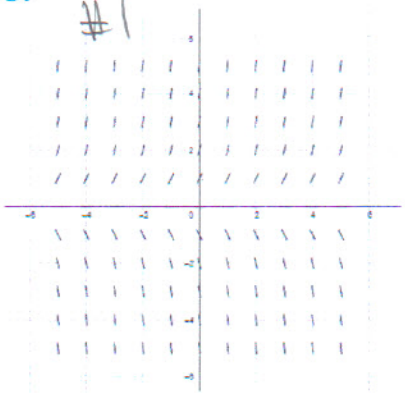
B #4



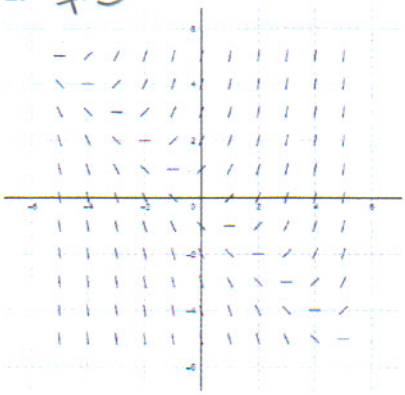
C. #2



D. #1



E. #5



F. #6