Calculus
Lesson 5.5: Bases other Than e and Applications
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The base of the natural exponential function is e. This "natural" base can be used to assign a meaning to a general base a.

DEFINITION OF EXPONENTIAL FUNCTION TO BASE *a*

If a is a positive real number $(a \neq 1)$ and x is any real number, then the **exponential function to the base *a*** is denoted by a^x and is defined by

$$a^x = e^{(\ln a)x}.$$

If $a = 1$, then $y = 1^x = 1$ is a constant function.

Radioactive Half-Life Model

- The half-life of carbon-14 is about 5715 years. A sample contains 1 gram of carbo-14. How much will be present in 10000 years?

DEFINITION OF LOGARITHMIC FUNCTION TO BASE *a*

If a is a positive real number $(a \neq 1)$ and x is any positive real number, then the **logarithmic function to the base *a*** is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{1}{\ln a} \ln x.$$

(change of base formula:
$$\log_a x = \frac{\log_b x}{\log_b a})$$

PROPERTIES OF INVERSE FUNCTIONS

1. $y = a^x$ if and only if $x = \log_a y$
2. $a^{\log_a x} = x$, for $x > 0$
3. $\log_a a^x = x$, for all x

Bases Other Than e

- Solve for x in each equation.

a. $3^x = \frac{1}{81}$

b. $\log_2 x = -4$

THEOREM 5.13 DERIVATIVES FOR BASES OTHER THAN *e*

Let a be a positive real number ($a \neq 1$) and let u be a differentiable function of x.

1. $\frac{d}{dx}[a^x] = (\ln a)a^x$
2. $\frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$
3. $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$

Differentiating Functions to Other Bases

- Find the derivative of each function.

a. $y = 2^x$

b. $y = 2^{3x}$

c. $y = \log_{10} \cos x$

Integrals for Bases Other than e

$$\boxed{\int a^x dx = \left(\frac{1}{\ln a}\right)a^x + C}$$

$\int 2^x dx$

THEOREM 5.14 THE POWER RULE FOR REAL EXPONENTS

Let n be any real number and let u be a differentiable function of x.

1. $\frac{d}{dx}[x^n] = nx^{n-1}$
2. $\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$

Comparing Variables and Constants

$\frac{d}{dx}[e^e]$

$\frac{d}{dx}[e^x]$

$\frac{d}{dx}[x^e]$

$y = x^x$

Compound Interest Formulas

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

Comparing Continuous and Quarterly Compounding

- A deposit of $2500 is made in an account that pays an annual interest rate of 5%. Find the balance in the account at the end of 5 years if the interest is compounded a) quarterly, b) monthly, and c) continuously.

Bacterial Culture Growth

- A bacterial culture is growing according to the logistic growth function

$$y = \frac{1.25}{1 + 0.25e^{-0.4t}} \quad t \geq 0$$

where y is the weight of the culture in grams and t is the time in hours. Find the weight of the culture after a) 0 hours, b) 1 hour, c) 10 hours. d) What is the limit as t approaches infinity.