Precalculus
Lesson 12.2: Arithmetic Sequences
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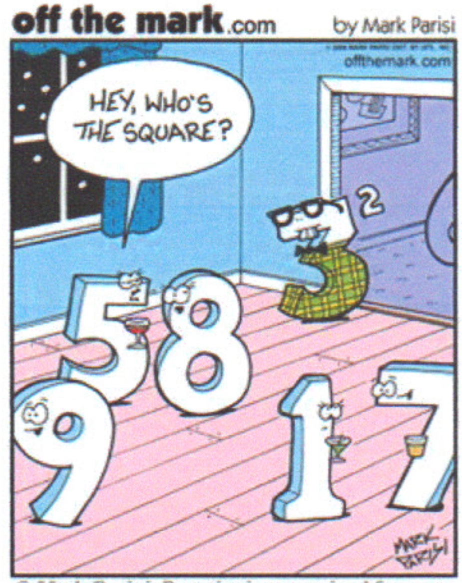

When any two numbers in a sequence differ by a constant value, the sequence is identified as an **Arithmetic Sequence.**

An arithmetic sequence may be **defined recursively** as:

$$a_1 = a, \qquad a_n - a_{n-1} = d$$

1st term

difference of 2 consecutive terms

For an arithmetic sequence $\{a_n\}$ whose first term is $\boldsymbol{a_1}$ and common difference is ***d***, the ***nth*** **term** is determined by the formula:

$$a_n = a_{n-1} + d$$ ← memorize

Determine if the sequence is Arithmetic, what is the common difference?

4, 6, 8, 10 ....
2 2 2

yes arithmetic – difference is constant
$d = 2$

$\{s_n\} = \{3n + 5\}$

$S_1 = 3(1)+5 = 8$ 〉 $d=3$
$S_2 = 3(2)+5 = 11$ 〉 $d=3$
$S_3 = 3(3)+5 = 14$

arithmetic with $d = 3$

$\{t_n\} = \{4 - n\}$

$t_1 = 4-1 = 3$ 〉 $d=-1$
$t_2 = 4-2 = 2$ 〉 $d=-1$
$t_3 = 4-3 = 1$

yes, arithmetic
$d = 1$

Finding the formula for an Arithmetic Sequence:

*n*th Term of an Arithmetic Sequence

For an arithmetic sequence $\{a_n\}$ whose first term is a_1 and whose common difference is d, the nth term is determined by the formula

$$a_n = a_1 + (n-1)d \leftarrow \text{Memorize}$$

Find the forty-first term of the arithmetic sequence: 2, 6, 10, 14, 18, ...

$n = 41$

$d = 6 - 2 = 4$

$a_1 = 2$

$a_{41} = 2 + (41-1)(4)$

$= 2 + 40(4)$

$= 2 + 160 = \underline{162}$

Finding the Recursive Formula for an Arithmetic Sequence:

The 8th term of an arithmetic sequence is 75, and the 20th term is 39.

a) Find the first term and the common difference
b) Give a recursive formula for the sequence.
c) What is the nth term of the sequence?

$a_8 = 75$

$a_{20} = 39$

write terms in n^{th} term formula:

(a)

$75 = a_1 + (8-1)d \rightarrow \overset{(-1)}{75} = \overset{(-1)}{a_1} + \overset{(-1)}{7d}$

$39 = a_1 + (20-1)d \rightarrow 39 = a_1 + 19d$

$-36 = 12d$

$-36/12 = \underline{d = -3}$

$75 = a_1 + (7)(-3)$

$75 + 21 = \underline{a_1 = 96}$

(b) $a_1 = 96,\ a_n = a_{n-1} - 3$

(c) $a_n = 96 + (n-1)(-3)$

$a_n = 96 - 3n + 3$

$a_n = 99 - 3n \leftarrow$ Simplified form

Finding the Sum of an Arithmetic Sequence

The sum of the first n terms of an arithmetic sequence is known as a **Partial Sum of an Arithmetic Sequence**

Let $\{a_n\}$ be an arithmetic sequence with first term a_1 and common difference of d. The sum S_n of the first n terms of $\{a_n\}$ may be found in two ways:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$= \sum_{k=1}^{n} [a_1 + (k-1)d] =$$ ← The sum of the first n terms

OR

#1 $$\boldsymbol{S_n = \frac{n}{2}[2a_1 + (n-1)d]}$$

#2 $$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

2 different formulas using 2 different formulas:

Find the sum S_n of the first n terms of the sequence: $\{a_n\} = \{\underbrace{3n+5}_{n^{th}\text{ term}}\}$; $a_1 = 3(1)+5 = 8$

$a_2 = 6+5 = 11$

$d = 3$

#1

$S_n = \frac{n}{2}(2(8)+(n-1)3)$

$= \frac{n}{2}(16+3n-3)$

$S_n = \frac{n}{2}(13+3n)$

#2 $S_n = \frac{n}{2}(8+3n+5)$

$S_n = \frac{n}{2}(13+3n)$

either formula works!

Find the sum: $60 + 64 + 68 + 72 + \cdots + 120$

$a_1 = 60$, $a_n = 120$ $d = 4$

both formulas need "n" use n^{th} term formula

$a_n = a_1 + (n-1)d$

$120 = 60 + (n-1)(4)$

$-60 \quad -60$

$60 = 4n - 4$

$64 = 4n$

$n = 16$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{16}{2}(60+120)$

$= 8(180$

$S_n = 1440$