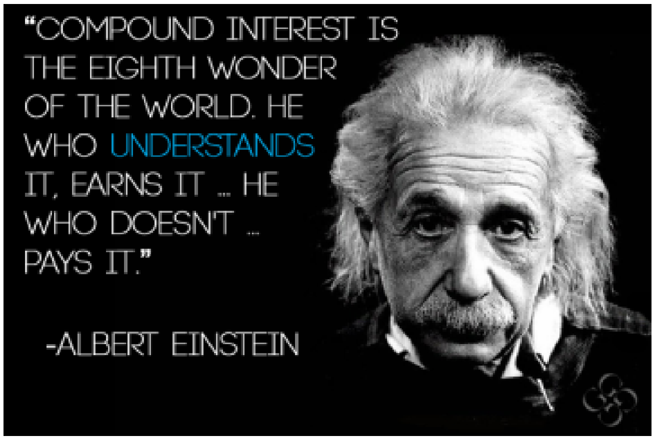
Precalculus

Lesson 5.7: Financial Models

Mrs. Snow, Instructor



Interest is the money paid for the use of money. Money borrowed is called **principal.** When you borrow money there is a **rate of interest**, expressed as a percent is charged over the amount of time of the loan. Most often the loan is compounded a number of times per year.

Compound Interest

Compound interest is calculated by the formula:

$$\boldsymbol{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$$

- A(t)= amount after t years
- P=Principal
- r=interest rate per year
- n=number of times compounded per year
- t=number of years

Calculate and compare the amount of money after one year using different compounding periods.

How much money will you have after one year, if you invest $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly, and daily?

<u>Continuously</u> Compounded Interest

Continuously compounded interest uses the base $\boldsymbol{e}$ and is calculated by the formula: $$\boldsymbol{A(t) = Pe^{rt}}$$ A(t)=amount after t years P=Principal r=interest rate per year t=number of years
Find the amount after 1 year if a principal investment of $1000 is invested at an interest rate of 10% per year, compounded continuously
What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

Precalculus

Lesson 5.8: Exponential Growth and Decay Models: Newton's Law, Logistic Growth and Decay

Mrs. Snow, Instructor

Many natural phenomena have been found to follow the law that an amount N varies with time t according to the function. Here we have the **Exponential Growth Model**

$$N(t) = N_0 e^{kt} \qquad k > 0$$

N_0 =the original amount
t = time
k =constant that represents the growth rate

Bacterial Growth

A colony of bacteria grows according to the law of uninhibited growth according to the function: $N(t) = 100e^{0.045t}$

Where N is measured in grams and t is measured in days.

a. Determine the initial amount of bacteria.
b. What is the growth rate of the bacteria?
c. Graph the function.
d. What is the population after 5 days?
e. How long will it take for the population to reach 140 grams?
f. What is the doubling time for the population?

Uninhibited Radioactive Decay

The growth/decay formula may be applied to radioactive decay:

The amount A of a radioactive material present at time t is given by:

$$A(t) = A_0 e^{kt} \qquad k < 0$$

A_0 =the original amount of radioactive material
k = a negative number that represents the rate of decay
t =time

Estimating the Age of Ancient Tools

Traces of burned wood along with ancient stone tools in an archeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon 14.

- The half-life of carbon 14 is 5700 years.
    a. Approximately when was the tree cut and burned?
    b. Graph the relation between the percentage of carbon 14 remaining and time.

Newton's Law of Cooling

Newton's Law of Cooling stated that the temperature of a heated object decreases exponentially over time toward the temperature of the surrounding medium:

The temperature u of a heated object at a given time t can be modeled by:

$$u(t) = T + (u_0 - T)e^{kt} \qquad k < 0$$

$u(t)$ = temperature
T =surrounding temperature
u_0 =initial temperature
k = constant (negative number)
t =time

Using Newton's Law of Cooling

An object is heated to $100°$and is then allowed to cool in a room whose air temperature is $30°C$.

- If the temperature of the object is $80°C$after 5 minutes, when will its temperature be $50°C$?
- Graph the relation found between the temperature and time
- Determine the elapsed time before the object is $35°C$

Logistic Model

If we look back at the growth model for cell, we realize that the formula allows for unlimited growth. We know that cell division eventually is limited by factors such as living space and food supply. This idea is called **carrying capacity.** The **logistic model** may be used to describe situations where the growth or decay of the dependent variable is limited.

The population P after time t is given by:

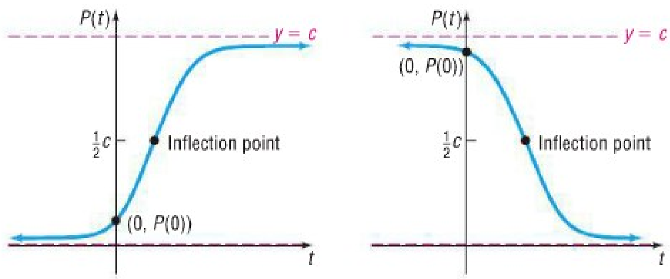
$$P(t) = \frac{c}{1 + ae^{-bt}}$$

$a, b, and\ c$ = constants with $a > 0\ and\ c > 0$

$b > 0$ growth

$b < 0$ decay

The general graph looks like:



Properties of the Logistic Model, Equation (5)

1. The domain is the set of all real numbers. The range is the interval $(0, c)$, where c is the carrying capacity.
2. There are no x-intercepts; the y-intercept is $P(0)$.
3. There are two horizontal asymptotes: $y = 0$ and $y = c$.
4. $P(t)$ is an increasing function if $b > 0$ and a decreasing function if $b < 0$.
5. There is an **inflection point** where $P(t)$ equals $\frac{1}{2}$ of the carrying capacity.

   The inflection point is the point on the graph where the graph changes from being curved upward to curved downward for growth functions and the point where the graph changes from being curved downward to curved upward for decay functions.
6. The graph is smooth and continuous, with no corners or gaps.

Fruit Fly Population

Fruit flies are placed in a half-pint milk bottle with a banana (for food) and yeast plants (for food and to provide a stimulus to lay eggs). Suppose that the fruit fly population after t days is given by:

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

a. State the carrying capacity and the growth rate.
b. Determine the initial population.
c. What is the population after 5 days?
d. How long does it take for the population to reach 180?
e. Determine how long it takes for the population to reach one-half of the carrying capacity.