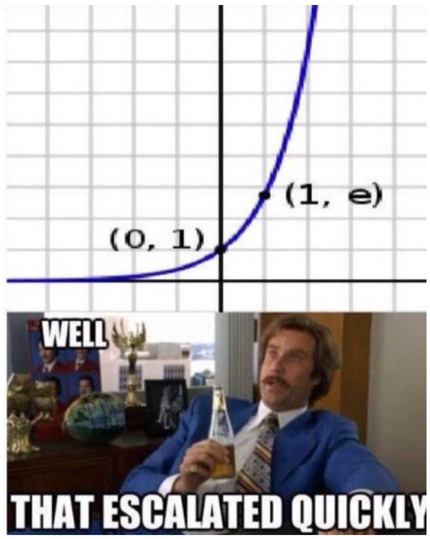
Calculus
Lesson 5.4 Exponential Functions: Differentiation and Integration
Mrs. Snow, Instructor



As we learned back in Algebra II, logarithm functions and exponential functions are inverses of one another. As Algebra II was a long time ago, let's do a quick review.

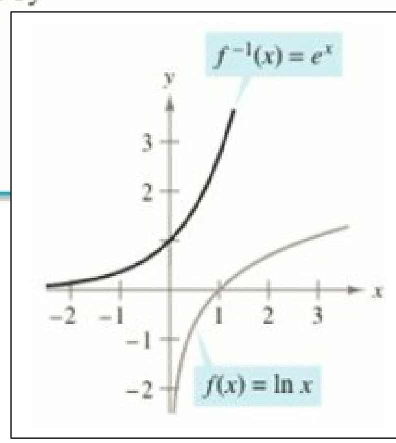
DEFINITION OF THE NATURAL EXPONENTIAL FUNCTION

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \quad \text{if and only if} \quad x = \ln y.$$



$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

Solving Exponential and Logarithmic Equations

$7 = e^{x+1}$

$\ln(2x - 3) = 5$

THEOREM 5.10 OPERATIONS WITH EXPONENTIAL FUNCTIONS

Let a and b be any real numbers.

1. $e^a e^b = e^{a+b}$
2. $\dfrac{e^a}{e^b} = e^{a-b}$

PROPERTIES OF THE NATURAL EXPONENTIAL FUNCTION

1. The domain of $f(x) = e^x$ is $(-\infty, \infty)$, and the range is $(0, \infty)$.
2. The function $f(x) = e^x$ is continuous, increasing, and one-to-one on its entire domain.
3. The graph of $f(x) = e^x$ is concave upward on its entire domain.
4. $\lim\limits_{x\to-\infty} e^x = 0$ and $\lim\limits_{x\to\infty} e^x = \infty$

The natural exponential function is its own derivative!!

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x.

1. $\dfrac{d}{dx}[e^x] = e^x$
2. $\dfrac{d}{dx}[e^u] = e^u \dfrac{du}{dx}$

a. $\dfrac{d}{dx}[e^{2x-1}]$

b. $\dfrac{d}{dx}[e^{-3/x}]$

Locating Relative Extrema

$f(x) = xe^x$

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x\,dx = e^x + C$ **2.** $\int e^u\,du = e^u + C$

Integrating Exponential Functions

Find $\int e^{3x+1}\,dx$.

Find $\int 5xe^{-x^2}\,dx$.

Integrating Exponential Functions

a. $\int \frac{e^{1/x}}{x^2}\,dx$

b. $\int \sin x\, e^{\cos x}\,dx$

Finding Areas Bounded By Exponential Functions

a. $\int_0^1 e^{-x}\,dx$

b. $\int_0^1 \frac{e^x}{1+e^x}\,dx$

c. $\int_{-1}^0 [e^x\cos(e^x)]\,dx$